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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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THE VISIBILITY OF RADIATION

By EDWARD P. HYDE, W. E. FORSYTHE, AND F. E. CADY

It would almost appear redundant, in view of the numerous and careful determinations of the visibility of radiation, such as those of Ives,[1] Nutting,[2] and Coblentz and Emerson,[3] to undertake a new investigation of this function. And yet a casual survey of the literature reveals striking differences among the results of the several investigators, and in the second place the old and much-mooted question of the reliability of the highly developed method of flicker photometry in giving results consistent with the older and more commonly used method of direct comparison is apparently still unsettled.

In connection with a new experimental investigation of the relative brightness of a black body as a function of its temperature, in which the measurements of brightness were made by the method of direct comparison, it was desired to compare the relative brightnesses as determined experimentally against the corresponding relative values computed from the energy-curves as given by Planck's equation and visibility data, after the method of Eisler.[4]

[1] *Phil. Mag.* (6), **24**, 149, 1912, and *Phys. Rev.*, **6**, 329, 1915.

[2] *Trans. Illum. Eng. Soc.* (U.S.), **9**, 633, 1914, and **13**, 108, 1918.

[3] *Scientific Papers of the Bureau of Standards*, No. 303; *Bulletin of the Bureau of Standards*, **14**, 167, 1918.

[4] *Elek. Zeit.*, **25**, 188, 1904.

To this end it seemed advisable to undertake a new determination of the visibility-curve by the method of direct comparison and under experimental conditions which should be so chosen as to be comparable with those involved in the measurements of brightness rather than to accept any of the published flicker values, or to impose upon the new determination any of the various specified conditions suggested by the flicker investigations but at variance with the conditions obtaining in the measurements of brightness of the black body, e.g., the restriction of the size of the field such as has become common practice in flicker photometry.

It is not the intention in this paper to enter upon a historical discussion of the subject; for this the reader is referred to the papers of other investigators, notably to the recent extended monograph on the subject by Coblentz and Emerson. It is of great importance, however, for the present purpose to consider briefly the question of the relative merits of the method of flicker photometry and of the older method of direct comparison in equating illuminations of different color.

It is a matter of opinion on what basis two illuminations of different color should be adjudged equal, and yet it is probable that by consensus of opinion equality of brightness as given in an ordinary photometer would be selected as the criterion, granted, of course, that this judgment can be formed. Herein lies the whole difficulty, and because it was found that illuminations of gross difference in color could only approximately and with great difficulty be equated in intensity by the method of direct comparison, attention was turned to the flicker method, which permits of comparatively easy measurements, even though individual peculiarities still exhibit themselves. And when it was thought to have been found that the visibility of radiation as determined by the flicker method was sensibly the same as that indicated by the less accurate method of direct comparison, exponents of the flicker method arose who wished to standardize this method as the accepted one for all heterochromatic measurements.

The two questions which the authors wish to raise are: (1) whether the inaccuracies of the direct-comparison method in all ordinary practical problems of heterochromatic photometry

are so large as to demand the introduction of a new method; and (2) whether the findings of the new method, under the prescribed experimental conditions, are the same as those of the older and commonly accepted method within the errors of measurement.

In answer to the first question the authors would refer to the paper by Middlekauff and Skogland[1] on "An Interlaboratory Photometric Comparison of Glass Screens and of Tungsten Lamps, Involving Color Differences." This paper reports the results of measurements made at several laboratories on the transmission of various blue-glass screens and on the relative candle-powers of several tungsten lamps each operated at a number of widely different voltages. The report shows that in the determination of the relative candle-powers of tungsten lamps at 72 volts and 132 volts, involving a color-difference of the order of magnitude of that existing between an old-type 4 w.p.c. carbon lamp and a 0.85 w.p.c. vacuum tungsten lamp, three laboratories using the Lummer-Brodhun photometer obtained results agreeing among themselves within a maximum difference of less than 2 per cent. And, as will be pointed out later, at least a part of this difference is probably to be ascribed to the small number of observers at each laboratory and the consequent undue weight assigned to individual idiosyncrasies of vision. If a determination involving so large a color-difference can be made by the older method with an error of probably less than 1 per cent from the mean, there would seem to be little need of introducing a new method, particularly if the foundation upon which it rests is insecure. In this same comparison a determination at another laboratory with the flicker photometer gave results markedly different from the results of direct comparison, and a similar difference in the same direction and of even greater magnitude was indicated by the results of Crittenden and Richtmyer,[2] again using the flicker photometer.

The second question is partially answered in the discussion above. The results obtained in the intercomparison among the

[1] *Trans. Illum. Eng. Soc.* (U.S.), **11**, 164, 1916; *Bulletin of the Bureau of Standards*, **13**, 287, 1918.

[2] *Scientific Papers of the Bureau of Standards*, No. 299; *Bulletin of the Bureau of Standards*, **14**, 87, 1918.

various laboratories and also the results of a comparative study by Crittenden and Richtmyer show that ratios given by the flicker photometer, under the prescribed conditions of size of field, etc., are distinctly different from those found by the more commonly used method of direct comparison. For the color-difference referred to in the preceding paragraph the largest deviation of any laboratory using the direct-comparison method from the mean value for the three laboratories was 1 per cent, and, as already stated, the deviations from the mean would probably be lessened if the readings of a larger number of observers in each laboratory should be taken. On the other hand, the values obtained with the flicker photometer in two different laboratories and with no strictures on account of the limited number of observers are, on the average, about 2.5 per cent different from the mean value found by the other method. The direction of the difference is such as to indicate that the less refrangible end of the spectrum is given relatively more weight in the flicker method, so that the relative candle-power of a lamp at any temperature in terms of its candle-power at a lower temperature is found to be smaller than that obtained by the method of direct comparison.

It is true that with the method of flicker photometry employed certain limitations with regard to size of field were prescribed which, it might be argued, account, at least in part, for the observed difference, but other data are available which would seem to vitiate this explanation as a complete one. Luckiesh[1] performed an experiment in which two fields, one red and the other blue-green, were compared by both the flicker and the direct-comparison methods, using the same apparatus. He found for his eye that the ratio blue-green to red was very much larger (50 to 100 per cent) with the direct-comparison method as compared with that obtained with the flicker method.

The recent elaborate investigation by Coblentz and Emerson[2] on visibility also indicates relatively greater blue sensibility in the method of direct comparison, but strangely the results of Ives[3] and of Coblentz and Emerson are at variance in one very important aspect. Whereas the data of Coblentz and Emerson show that the

[1] *Electrical World*, **61**, 620 and 835, 1913. [2] *Loc. cit.* [3] *Loc. cit.*

visibility-curve obtained with the method of direct comparison is somewhat broader and flatter than the corresponding curve obtained with the flicker method, the data of Ives point to the opposite conclusion—a result markedly indicated by the experimental data to be presented later in this paper. The importance of this difference lies in its effect on the resultant computed value for the mechanical equivalent of light, since a difference in the area of the visibility-curve affects directly the value of this important constant.

In view of the foregoing considerations it would seem to the authors that somewhat different answers must be given to the two questions raised above from those which the advocates of the method of flicker photometry are urging. Whatever may be the uncertainties in determining the average visibility-curve by the method of direct comparison (a question on which the authors will later adduce evidence), there is much reason to believe that for those color-differences which are commonly encountered in practical photometry the method of direct comparison may be used with reasonable confidence. And secondly, the evidence available seems to show that the flicker method, as commonly employed, does not yield results consistent with those obtained by the older method, and that the differences between the two are sufficiently pronounced to manifest themselves both in the ordinary heterochromatic measurements of practical photometry and in the curves of visibility obtained by the two methods.

Before describing the method and presenting the results of the present investigation the authors wish to point out that the dominant thought in the investigation was to reproduce in the determination of visibility the conditions obtaining in ordinary photometry and to pay no special attention to some of the minor conditions which might have been imposed as a result of the recent investigations on the subject. Thus no attempt was made to keep the illumination constant, and so the brightness at the ends of the spectrum was much lower than that in the more luminous regions. At 0.5μ the brightness of the Lummer-Brodhun cube was approximately 0.001 candles per cm^2, at 0.56μ it was 0.005 candles per cm^2, and at 0.65μ it was 0.003 candles per cm^2. These

brightnesses correspond approximately to illuminations of a perfectly reflecting, perfectly diffusing surface of 30, 150, and 90 meter candles. Since an artificial pupil of $0.6\ mm^2$ was employed, the illumination intensities of the retina would be somewhat smaller than those corresponding to the same objective brightnesses in practical photometry. It is seen that everywhere over the range of wave-length investigated the brightness was reasonably high and probably beyond that of the Purkinjé region except possibly at the extreme blue end of the spectrum. Moreover, the authors do not think the evidence at present available sufficient to justify the conclusion that the visibility-curve obtained under the conditions of equal brightness is to be preferred to that obtained under the normal conditions of a dispersed spectrum, granted the two are different.

APPARATUS AND METHOD

The distinguishing characteristics of the present investigation, apart from the employment of the method of direct comparison under conditions with respect to size of field, etc., obtaining in ordinary photometric practice, are to be found in the use of the step-by-step method and in the determination of the distribution of energy in the spectrum. The step-by-step method has been employed before, as in an experiment by Ives,[1] but to the best of the authors' knowledge this method has not been used in any extended investigation with a large number of observers. The steps were chosen so small (varying from $0.0052\,\mu$ in the red [$\lambda=0.66\,\mu$] to $0.0022\,\mu$ in the blue [$\lambda=0.5\,\mu$]) that on the basis of Steindler's[2] data the interval everywhere throughout the spectrum would be less than that corresponding to the limen of hue-discrimination. It was subsequently found, when the apparatus had been constructed and the experiment begun, that the step was still too large to eliminate all hue-differences, though these differences were relatively small in magnitude. As it was, some fifty steps were required to span the range of spectrum studied, though the method of investigation was such as to avoid the necessity of actually making so many measurements.

[1] *Loc. cit.*

[2] *Wiener Sitzungsberichte* (II*a*), **115**, 1, 1906.

The evaluation of the spectral energy was founded on the determination of the color-temperature of the source. The color-temperature is the temperature of a black body having the same distribution of energy in the visible spectrum as the source employed. By means of Planck's equation the energy-distribution was computed and, allowing for dispersion and absorption by the optical system and for scattering, the relative energy entering the eye in the different parts of the spectrum was readily determined. In the opinion of the authors this method has much to commend it

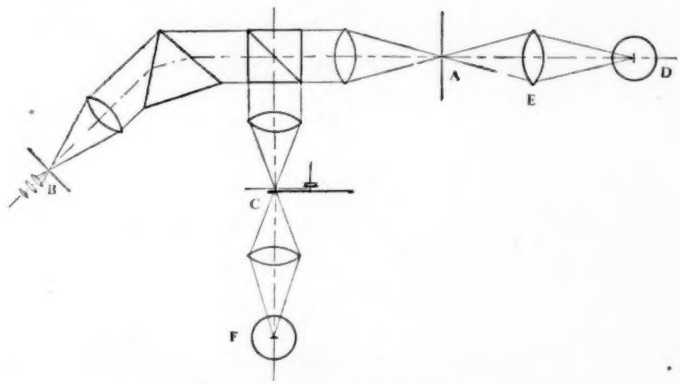

FIG. 1.—Diagram of apparatus

over the very difficult and uncertain method of attempting to measure directly the energy at the eye-slit.

The apparatus employed (Fig. 1) consisted of a tungsten lamp *D*, whose broad, flat filament was focused by means of the projection lens *E* on the slit *A* of a Lummer-Brodhun spectrophotometer, having the absorption strips removed from the Lummer-Brodhun prism so that the settings were made on the basis of equal brightness rather than on that of equal contrast. The comparison field was obtained from a second tungsten lamp *F*, the settings being made by means of the special variable rotating sectored disk *C*.[1] A low-power eyepiece *B* was employed in order to facilitate fixation upon the diagonal surface of the Lummer-Brodhun cube where are located the two fields to be compared.

[1] *Astrophysical Journal*, **35**, 237, 1912.

The slit *A* was specially designed so that it could be moved sidewise a definite fixed amount, 0.15 mm, thus providing the means of securing a small shift of one spectrum with respect to the other. The amount of this shift, expressed in wave-lengths and differing slightly in magnitude from one part of the spectrum to another, was determined very carefully in several different ways.

The test lamp *D* was operated at a color-temperature of 2045° K maintained constant throughout the experiments. It was so mounted that it could be moved with the slit. In this way the

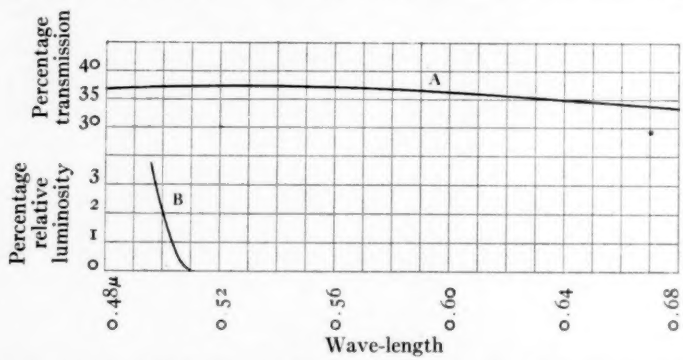

FIG. 2.—Corrections for stray light and selective transmission of optical system. A, selective transmission of optical system; B, luminosity of stray light at wave-lengths shown, in percentage of total luminosity at the corresponding wave-lengths.

same portion of the broad filament was always focused on the slit, and consequently there was no alteration in radiation passing through it. The width of the slit *A* was 0.1 mm and that of the ocular slit 0.2 mm, being thus so small as to produce an error of only a fraction of 1 per cent owing to the impurity of the spectrum.[1] Of course it was necessary to allow for the dispersion of the prism in computing the energy-distribution at the eyepiece, and to correct for stray light and for the selective absorption of the complete optical system.

These corrections are shown in Fig. 2. The stray light was evaluated through the use of color-screens placed in front of the eyepiece slit, and was determined directly in units of luminosity rather than of energy. It is seen to be a relatively small correc-

[1] *Astrophysical Journal*, **35**, 237, 1912.

tion factor. The selective absorption of the optical system was obtained through the aid of a spectral pyrometer and was determined in two different ways yielding consistent results.

In carrying out the main experiment the method employed consisted in placing the slit A in its middle or normal position and varying the voltage of the comparison source F until the relative brightnesses of the two photometric fields were approximately the same throughout the spectrum. Then a set of measurements was made of the relative brightnesses at eighteen different wave-lengths distributed at approximately equal wave-length intervals from $0.5\,\mu$ to $0.66\,\mu$, the range of wave-lengths studied. Then the slit A was displaced laterally by the predetermined fixed amount and the same set of measurements repeated. If

$$R_1=\frac{L'_\lambda}{L_\lambda}$$

is the observed ratio of the luminosity (L'_λ) of the comparison field at the wave-length λ to that (L_λ) of the test field at the same wave-length (as determined by the first experiment), and

$$R_2=\frac{L'_\lambda}{L_{\lambda+\Delta\lambda}}$$

is the observed ratio of the luminosity (L'_λ) of the comparison field at the wave-length λ to that ($L_{\lambda+\Delta\lambda}$) of the test field at the wave-length $\lambda+\Delta\lambda$ (as determined by the second experiment, when the slit A has been shifted an amount corresponding to $\Delta\lambda$), then the ratio

$$R_\lambda=\frac{L_\lambda}{L_{\lambda+\Delta\lambda}}$$

of the two luminosities of the test field at the wave-lengths λ and $\lambda+\Delta\lambda$ is seen to equal R_2/R_1, and so is determinable from the two sets of measurements.

If now these experimentally determined values of R_λ at the eighteen points throughout the spectrum are plotted against the corresponding wave-lengths, a curve may be drawn giving the value of R_λ for the interval $\Delta\lambda$ for every wave-length. Then starting at one end of the spectrum and proceeding by successive

intervals $\Delta\lambda$ (differing slightly in different parts of the spectrum, as previously determined), the relative luminosity-curve of the test field for the observer making the measurements is computed by multiplying the observed ratio R_{λ_1} corresponding to the interval $\Delta_1\lambda$ by the ratio R_{λ_2} corresponding to the next interval $\Delta_2\lambda$, and this product in turn by R_{λ_3} corresponding to the next interval, and so on until the other end of the spectrum has been reached, plotting the value of the product at each successive step as the relative luminosity of the test field at the corresponding wave-length.

It is evident that since the number of these steps is determined by the magnitude of the displacement of the slit A this quantity must be known with accuracy. The uncertainty in the value of this quantity is so small as to produce a probable error in the final luminosity-curve of not more than 3 or 4 per cent, which is less than uncertainties arising from other sources.

Each of twenty-nine observers, most of whom were experienced in photometric measurements, made at least two independent sets of determinations by this method, and the average of the several sets of any observer was taken as giving his luminosity-curve for the energy-distribution employed. Some observers obtained remarkably consistent results (within 3 or 4 per cent) in their independent sets of measurements, while others showed differences several times as large.

The twenty-nine luminosity-curves thus obtained were then averaged, employing a method of averaging somewhat different from either of the two methods that have been used in other recent investigations. The luminosity-curves were all reduced to the same area and then the ordinates averaged at each wave-length, taken in steps of $0.01\,\mu$. This method would seem, in the judgment of the authors, to have a better theoretical basis than that of averaging the ordinates of the individual visibility-curves reduced to the same area, or that of averaging the ordinates of the individual visibility-curves reduced so that the maximum ordinate of each is unity.

The reason for the adoption of this method is as follows: The integral luminous flux from a source at any given color-temperature should be assumed to be the same for all observers, and the weight

assigned to the luminosity in any region of the spectrum for any observer should be determined on the basis of equal total luminous flux for that color-temperature. This weighting would of course be different for different energy-distributions corresponding to various color-temperatures of the source, but if some average temperature is taken the resultant average visibility-curves will be entirely correct for that temperature and only slightly in error for other temperatures, since the range of color-temperature encountered in practical photometry is relatively small. Theoretically it is preferable to choose as the standard color-temperature that of the carbon lamps adopted as the representative standards of luminous intensity, but since the color-temperature of the source employed in the present investigation is so nearly that of the standard carbon lamps (2077° K), the results are practically the same as those which would have been obtained had correction been made to the latter. The color-temperature which we have taken is 2045° K, that of the test lamp *D*, or more accurately a slightly higher temperature corresponding to the actual distribution of energy at the ocular slit, which is slightly different from that of the source, owing to scattered light and selective absorption of the optical system. The same assumption probably underlies the method of averaging the individual visibility-curves reduced to the same area, but this corresponds to an equal energy-distribution throughout the spectrum and so lies entirely outside the range of experience.

The method of reducing the various individual visibility-curves to the same value of maximum ordinate would seem to have no theoretical foundation and must be judged by its results. For the twenty-nine curves obtained in the present investigation it was found that this method of averaging yielded an average visibility-curve differing by many per cent at some wave-lengths from the curve obtained by the more rigorous method employed. As a matter of interest the observations of Coblentz and Emerson, who reduced the visibility-curves to equal maximum ordinates, were worked up by the more rigorous method, and the results showed an average visibility-curve sensibly the same as that derived by the other method. Coblentz and Emerson justified their method on

the ground of the large number of observers, and their conclusions were evidently warranted, but comparisons of individuals or of small groups taken from their one hundred and twenty-five observers would be subject to possible error unless the more rigorous method of averaging were employed.

EXPERIMENTAL RESULTS

The relative visibility data for the twenty-nine individual observers are given in Table I. These data are reduced on the basis of equal areas of the luminosity-curves for the chosen color-temperature (approximately 2045° K). The relative average visibility data for the twenty-nine observers, obtained on this basis, and also the average values obtained on the basis of equal value (unity) for the maximum ordinate of each visibility-curve are included. The former are also given in Table II and Fig. 3, where for purposes of comparison the published results of the recent investigations of Ives, Nutting, Coblentz and Emerson, and Reeves,[1] all obtained by the flicker method, are also included. With the exception of the data of Reeves,[2] which for some unknown reason differ largely from the other data obtained by the flicker method, it is seen that the visibility-curve given by the authors and obtained by the method of direct comparison is relatively narrower and more suppressed in the red end of the spectrum than the curves obtained with the flicker photometer. Nutting's curve most nearly agrees with that of the authors, and if his original published data had been used instead of his modified data, based on a more recent determination of the energy-distribution in the spectrum of his acetylene-flame source, the agreement would have been even better and well within the experimental errors.

From Fig. 3 the wave-length of maximum visibility may be taken to be 0.556 μ in the present investigation as compared with 0.557 μ found by Coblentz and Emerson, but the authors feel that

[1] *Trans. Illum. Eng. Soc.* (U.S.), **13**, 101, 1918.

[2] The data of Reeves are reduced to the same basis of energy-distribution for the acetylene flame as that employed by Nutting in his final corrected values (kindly furnished by Dr. Nutting), since this acetylene-flame source was the same as that used by Nutting. The energy-distribution for this flame was determined by Coblentz and found to be the same as that published by Coblentz in his paper of 1916 on the subject.

this value is uncertain by 0.003 μ, owing to the limitations involved in drawing the curve. The question naturally arises whether the twenty-nine observers of the present investigation represent a

TABLE I

RELATIVE-VISIBILITY DATA FOR TWENTY-NINE OBSERVERS REDUCED TO EQUAL AREAS OF THE WAVE-LENGTH LUMINOSITY-CURVES FOR THE CHOSEN COLOR-TEMPERATURE (APPROXIMATELY 2045° K)

Wave-Length	Relative-Visibility Data								
	E.P.H.	F.E.C.	R.G.B.	C.F.S.	I.W.	M.L.	W.W	W.E.F.	A.G.W.
μ									
0.50...	316	236	393	242	324	413	237	193	132
0.51...	526	417	641	406	502	642	418	353	236
0.52...	758	619	921	581	658	846	600	533	355
0.53...	911	805	1143	722	787	1004	736	699	469
0.54...	977	973	1315	830	907	1121	840	831	580
0.55...	975	1054	1341	870	972	1145	880	889	651
0.56...	952	1085	1276	894	1004	1124	898	922	710
0.57...	895	1030	1103	882	980	1032	881	908	741
0.58...	810	914	877	840	907	887	834	856	748
0.59...	705	760	652	768	782	708	759	770	727
0.60...	591	596	464	666	625	534	652	659	678
0.61...	472	446	314	541	467	384	528	531	607
0.62...	356	318	207	407	331	265	399	399	514
0.63...	254	217	132	285	224	175	283	283	409
0.64...	166	138	80	181	143	108	185	184	302
0.65...	103	84	47	108	88	64	114	114	205
0.66...	57	46	25	57	50	34	64	63	119

Wave-Length	Relative-Visibility Data								
	P.W.C.	W.W.K.	C.N.	P.F.S.	E.J.E.	H.H.K.	G.H.M.	H.O.	N.L.
μ									
0.50...	225	407	435	407	297	396	324	488	242
0.51...	351	659	670	623	473	615	463	731	377
0.52...	478	922	866	826	660	819	594	931	512
0.53...	597	1126	1021	993	814	981	706	1061	624
0.54...	712	1255	1134	1092	932	1105	808	1118	721
0.55...	784	1244	1137	1073	974	1128	852	1090	772
0.56...	843	1162	1086	1007	992	1093	880	1033	805
0.57...	856	1022	986	909	961	1004	874	939	803
0.58...	822	857	854	803	877	877	835	828	774
0.59...	750	681	704	691	752	721	761	697	720
0.60...	660	514	550	568	607	558	647	553	643
0.61...	559	362	405	439	466	404	515	412	555
0.62...	448	240	279	319	344	278	383	287	455
0.63...	333	152	182	220	241	183	270	190	348
0.64...	226	89	109	138	156	112	177	115	247
0.65...	143	51	62	83	97	66	111	67	164
0.66...	81	26	32	44	54	36	63	35	97

TABLE I—*Continued*

Wave-Length	Relative-Visibility Data								
	T.P.	C.F.K.	L.L.M.	W.L.E.	E.T.F.	K.H.M.	A.F.B.	I.A.V.	G.B.
μ									
0.50...	206	471	324	366	223	407	187	561	435
0.51...	364	708	526	560	378	610	298	757	599
0.52...	532	936	729	727	553	808	428	914	726
0.53...	654	1081	910	863	729	980	561	1011	844
0.54...	738	1192	1040	958	884	1093	694	1078	965
0.55...	772	1218	1056	978	956	1126	778	1063	1016
0.56...	798	1196	1027	975	990	1127	851	1015	1032
0.57...	799	1093	956	931	965	1055	881	930	994
0.58...	774	933	857	849	892	917	862	825	909
0.59...	723	729	731	729	775	735	795	700	770
0.60...	649	523	591	591	637	550	689	561	606
0.61...	560	344	447	456	497	386	566	421	444
0.62...	456	212	319	337	366	260	440	297	306
0.63...	347	124	217	238	254	169	322	197	202
0.64...	243	67	137	158	162	103	216	121	124
0.65...	159	35	82	101	97	62	135	71	73
0.66...	90	16	45	60	51	34	74	37	40

Wave-Length	Relative-Visibility Data				
	O.B.	L.W.H.	Average	Same in Terms of Maximum Taken as Unity	Average on a Different Basis (See Text)
μ					
0.50...........	424	206	328	328	322
0.51...........	675	333	514	515	507
0.52...........	895	484	697	698	690
0.53...........	1030	667	846	847	839
0.54...........	1091	861	967	968	954
0.55...........	1078	964	995	996	992
0.56...........	1034	1007	994	995	995
0.57...........	953	979	943	944	953
0.58...........	848	901	854	855	868
0.59...........	716	784	734	735	751
0.60...........	568	645	599	600	617
0.61...........	420	502	464	464	483
0.62...........	291	370	341	341	358
0.63...........	192	258	238	238	252
0.64...........	117	166	154	154	164
0.65...........	68	101	95	95	102
0.66...........	36	54	52	52	56

fairly average eye. It is possible through the medium of the results of the interlaboratory comparison of Middlekauff and Skogland, referred to previously, to compare the twenty-nine observers here with the one hundred and twenty-five observers employed in the investigation of Coblentz and Emerson. The

five observers of this laboratory who determined the ratio of candle-power of the tungsten lamps at 132 and 72 volts, respectively, were all included in the present investigation, and computations show that for the foregoing candle-power ratio (i.e., for the corresponding color-difference) the twenty-nine observers would obtain a ratio 0.5 per cent less than that obtained by the five

TABLE II

COMPARATIVE RELATIVE-VISIBILITY DATA OF VARIOUS INVESTIGATORS

Wave-Length	Relative-Visibility Data				
	Hyde Forsythe Cady	Ives Kingsbury	Nutting	Coblentz Emerson	Reeves
μ					
0.50	0.328	0.318	0.314	0.316	0.275
.51	.515	.473	.456	.503	.474
.52	.698	.637	.646	.710	.686
.53	.847	.801	.815	.862	.841
.54	.968	.915	.925	.954	.935
.55	.996	.988	.986	.994	.993
.56	.995	.996	.995	.998	.985
.57	.944	.947	.949	.968	.935
.58	.855	.859	.871	.898	.836
.59	.735	.758	.762	.800	.710
.60	.600	.653	.634	.687	.580
.61	.464	.534	.498	.557	.446
.62	.341	.396	.368	.427	.319
.63	.238	.283	.268	.302	.214
.64	.154	.183	.166	.194	.140
.65	.095	.110	.105	.115	....
.66	.052	.068	.058	.0645	....

observers. Similarly, taking the observers at the Bureau of Standards who participated in both investigations, computations show that the one hundred and twenty-five observers would obtain a ratio possibly 0.2 or 0.3 per cent greater than that obtained by the eight observers who were employed in the measurements of Middlekauff and Skogland. Since, according to the report of this interlaboratory comparison, Nela Research Laboratory obtained a ratio 1.9 per cent greater than that obtained at the Bureau, it would follow that the twenty-nine observers here would have obtained a ratio only 1.2 or 1.1 per cent greater than the one hundred and twenty-five observers used in the investigation of Coblentz and Emerson.

This difference is so small, considering the unsatisfactory way in which the comparison was carried out, that it would be unsafe to draw any conclusion except that, so far as can be ascertained, the twenty-nine observers employed in the present investigation do

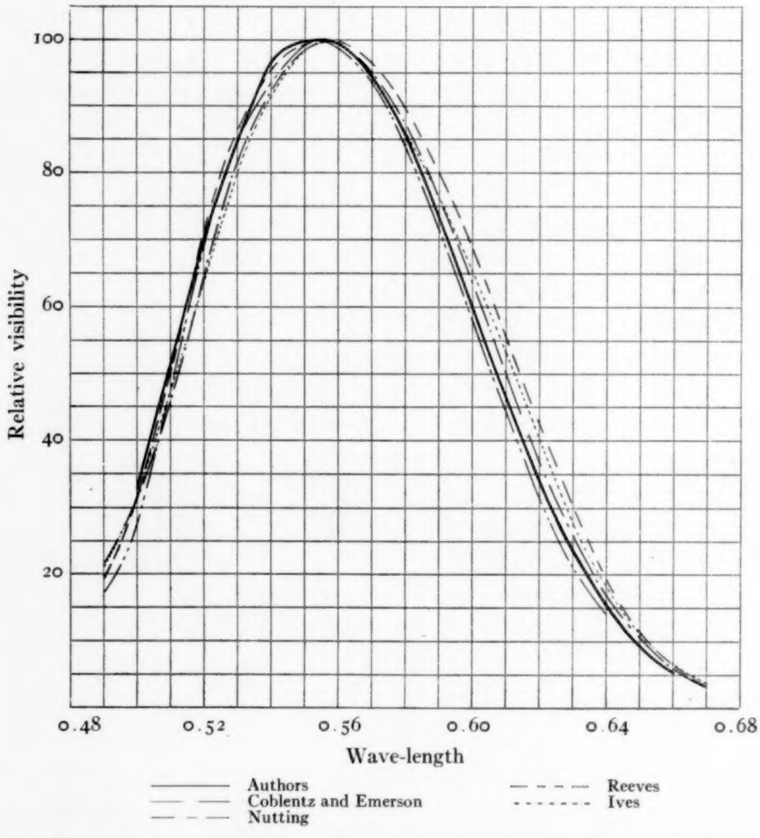

FIG. 3.—Relative average visibility data obtained in present investigation by the direct-comparison method as compared with published data of other investigators using the method of flicker photometry.

not differ materially in the average from the one hundred and twenty-five observers used in the investigation of Coblentz and Emerson. The comparison would seem to indicate, however, as intimated in an earlier paragraph, that the differences found by

Middlekauff and Skogland among the various laboratories would probably have been somewhat smaller had a larger number of observers been employed in each laboratory.

The significance of the difference between the visibility-curve obtained in the present investigation using the method of direct comparison and those obtained with the flicker method lies to some extent in the greater consistency of the results following the application of the former in computing relative candle-powers of a source at two widely different temperatures, but in large part in the distinctly smaller value of the mechanical equivalent of light which results from the use of the curve obtained by the method of direct comparison. As will be presented in a subsequent paper, the experimental values for the brightness of a black body at different temperatures are slightly more concordant with the computed values if the visibility-curve obtained here is used than with the results computed on the basis of the flicker-photometer visibility-curve, the latter giving relatively too much weight to the red end of the spectrum. And consequently the values of the mechanical equivalent of light computed from the brightnesses of the black body at different temperatures will agree among themselves if the visibility-curve obtained by the direct-comparison method is assumed, whereas this will not otherwise be true. These data will also be presented in the subsequent paper on the subject.

The considerations and data presented in this paper argue for the adherence to the older method of direct-comparison photometry in all ordinary practical work. As a means of avoiding the necessary difficulties in heterochromatic photometry in practice the authors refer to the proposal made years ago[1] that suitable color-screens be calibrated at the Bureau of Standards and distributed for use in photometric laboratories, so that comparisons involving large differences in color may be avoided in all except standardizing laboratories. It is true, however, that the results obtained with the flicker photometer are more consistent even over the comparatively small color intervals encountered in practical photometry, and the conditions of use have been thoroughly standardized, so that this instrument may find valuable application,

[1] *Electrical World*, **54**, 195, 1909.

particularly in a standardizing laboratory. But in the opinion of the authors the flicker scale should not supplant that of direct comparison, and consequently the visibility-curve obtained by the direct-comparison method should be used in computing luminosity-curves and relative candle-powers. Especially should the visibility-curve obtained by the method of direct comparison be used in the computation of the mechanical equivalent of light, in which its difference from that obtained by the flicker method is shown in the most pronounced way.

SUMMARY

1. In ordinary photometric comparisons involving difference of color the older method of direct comparison is adequate if a sufficient number of observers is employed to secure a fairly average eye.

2. The relative candle-power values found for sources of different color by the use of the flicker photometer are appreciably different from those obtained by the method of direct comparison. The flicker photometer apparently assigns too much weight to the red end of the spectrum.

3. The average curve of relative visibility for twenty-nine observers obtained by direct comparison, using a step-by-step method, is presented. Special attention is called to the method employed for determining the distribution of energy at the ocular slit.

4. The curve of relative visibility obtained by the method of direct comparison is found to be definitely different from that obtained by other investigators using the flicker method. The latter is somewhat broader and shows relatively too large values in the red end of the spectrum. Evidence will be presented in a subsequent paper to show the consistency between the newly determined visibility-curve and the findings of ordinary photometry in the case of the brightness of a black body at different temperatures.

5. Recommendation is made of the adherence to the older photometric method of direct comparison in practical photometry, and of the use of calibrated color-filters to simplify comparisons otherwise involving large difference in color.

APPENDIX I

In connection with the determination of the effective wave-length of transmission of so-called monochromatic red-glass screens used with optical pyrometers two of the present authors[1] some time ago described a determination of the relative visibility of radiation in the less refrangible end of the spectrum, using an adapted form of a spectral pyrometer. Owing to the large field-brightness obtainable with the pyrometer it was possible to carry the measurements farther into the red than had been possible before. Moreover, since the application of the data was to be made under conditions quite similar to those obtaining in their determination, it seemed unnecessary to enter into any elaborate discussion of these conditions or to question their applicability.

Subsequently L. W. Hartman,[2] working in this laboratory, employed the same method to extend the relative visibility data on the side of short wave-lengths, and incidentally again to furnish values which might be used in computing the effective wave-length of blue pyrometer glasses.

It therefore seemed advisable to one of the authors, in connection with the present investigation as presented in the body of this paper, to carry out a series of determinations of relative visibility with the pyrometer method through the central region of the visible spectrum, thus connecting the previously published data for the red and blue ends of the spectrum. A brief report of the results obtained is presented in this Appendix.

The method employed is identical with that already described in the earlier papers on the subject, except that an unsaturated green-glass screen in front of the eyepiece was used throughout in order to reduce the large differences in color and make more consistent settings possible. The transmission-curve for this screen, plotted in Fig. 4, shows a considerable transmission of light throughout the entire spectral region studied.

In this supplementary investigation, as in the principal one, the method of direct comparison was used, and the determination of the distribution of energy at the eyepiece was carried out in the

[1] *Astrophysical Journal*, **42**, 285, 1915. [2] *Ibid.*, **47**, 83, 1918.

same way. But in some important aspects the conditions in the two experiments were quite different. Thus the brightness in

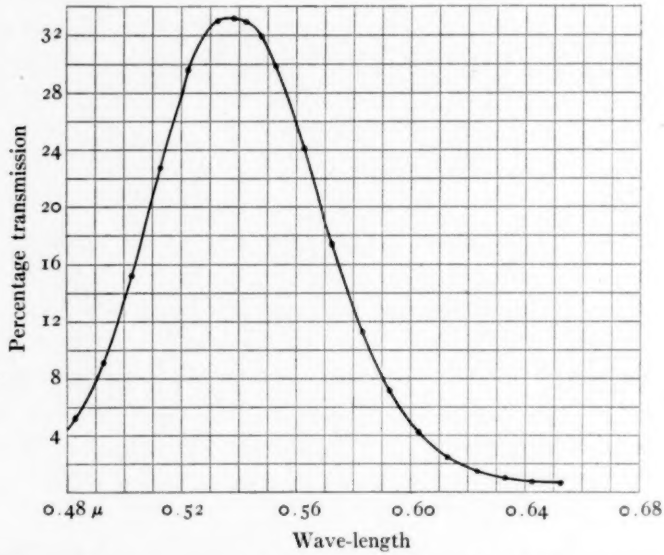

FIG. 4.—Selective transmission of green glass employed in eyepiece of spectral pyrometer.

TABLE III

RELATIVE AVERAGE-VISIBILITY DATA OBTAINED BY THE USE OF A SPECTRAL PYROMETER

(As compared with that obtained in the principal investigation in which the conditions, such as method employed, size of field, etc., were different)

WAVE-LENGTH	RELATIVE AVERAGE-VISIBILITY DATA		WAVE-LENGTH	RELATIVE AVERAGE-VISIBILITY DATA	
	* Spectral Pyrometer	† Spectro-photometer		Spectral Pyrometer	Spectro-photometer
μ			μ		
0.50........	0.329	0.328	0.59........	0.782	0.735
.51........	.552	.515	.60........	.639	.600
.52........	.736	.698	.61........	.489	.464
.53........	.886	.847	.62........	.347	.341
.54........	.958	.968	.63........	.229	.238
.55........	.993	.996	.64........	.161	.154
.56........	1.000	.995	.65........	....	.095
.57........	.999	.944	.66........	....	.052
.58........	.914	.855	....	....	....

the different spectral regions was always the same in the pyrometer experiments (0.024 candles per cm²), and the measurements were made throughout against a constant greenish comparison field. Moreover, the size of the field was much smaller, though

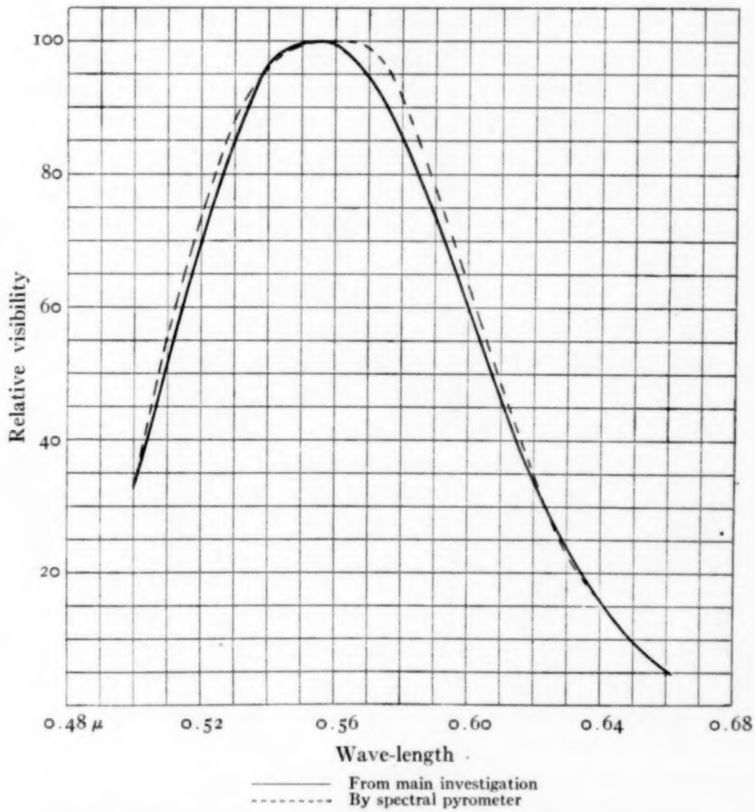

FIG. 5.—Relative average visibility data obtained by the use of a spectral pyrometer as compared with those obtained in the principal investigation in which the conditions, such as method employed, size of field, etc., were different.

the comparison field (pyrometer filament) was not so small as that employed in the earlier investigations of the extreme red and blue regions. The collimator slit was 0.3 mm, and the ocular slit was 0.12 mm. Corrections for slit-width were computed and found to be negligibly small.

Ten observers were employed, taken at random from among the twenty-nine used in the principal investigation, and each observer (with one or two exceptions) made at least two independent sets of measurements. The results, in comparison with those presented in the main body of the paper, are shown in Table III and Fig. 5. It is seen that the curve is somewhat flatter near the center and drops off more rapidly toward the two ends of the spectrum. The previously found visibility data in the red and blue ends of the spectrum, made by the same method, fit nicely on to the ends of this curve. The previously obtained data in the blue fit equally well on to the curve presented in the main body of this paper, but the data for the red end cannot be made to fit with any accuracy.

This curve agrees more closely with that found in the principal investigation than does any one of the curves obtained with the flicker method, and relative candle-powers of a black body at different temperatures computed on its assumption are reasonably satisfactory. Moreover, it conduces to approximately the same value of the mechanical equivalent of light as that calculated from the other curve. But owing to the inherent difficulties encountered in applying the pyrometer method in the middle region of the spectrum, to the difference between the conditions of the experiment and those of ordinary practice, and finally to the more limited number of observers, the authors do not attach as much weight to the results as to those presented in the body of the paper, particularly for application to cases of ordinary practical photometry.

APPENDIX II

The investigation described in the body of the present paper yielded a curve of relative visibility obtained by the method of direct comparison and extending from 0.50μ to 0.66μ. A knowledge of the curve over this interval is sufficient to compute relative candle-powers of a black body over a moderate range of temperature, such as from 1700° K to 2500° K, with errors amounting to only a few tenths of 1 per cent on account of the omission of the ends of the curve in the red and blue. If larger ranges of temperature are employed, the errors arising from this omission may be

appreciable, and at any temperature the application of the visibility data in computing the mechanical equivalent of light will conduce to erroneous values if the luminosities of the two ends of the spectrum are neglected. Finally there are problems, such as the determination of the effective wave-length of transmission of red- and blue-glass screens for use in optical pyrometry, in which a knowledge of the visibility data for the ends of the spectrum is necessary.

TABLE IV

RELATIVE-VISIBILITY DATA FOR ENTIRE SPECTRUM
(As recommended by authors)

Wave-Length	Relative Visibility	Wave-Length	Relative Visibility	Wave-Length	Relative Visibility
μ		μ		μ	
0.40	0.00009	0.52	0.698	0.64	0.154
.41	$.0006_2$	.53	.847	.65	.094
.42	$.004_1$	.54	.968	.66	.051
.43	$.011_5$	.55	.996	.67	.026
.44	.022	.56	.995	.68	$.012_5$
.45	.036	.57	.944	.69	$.006_2$
.46	.055	.58	.855	.70	$.003_1$
.47	.087	.59	.735	.71	.0015
.48	.138	.60	.600	.72	$.0007_4$
.49	.216	.61	.464	.73	$.0003_6$
.50	.328	.62	.341	.74	$.0001_8$
.51	.515	.63	.238	.75	.00009
				.76	.00005

* Average of 10 out of the 29 observers.
† Average of 29 observers.

It therefore appeared to the authors desirable to submit data on relative visibility covering practically the entire visible spectrum. The most probable values in the opinion of the authors are contained in Table IV.

These values from 0.50μ to 0.64μ are precisely the same as those given in the body of the paper. For the red end are chosen the published data of Hyde and Forsythe[1] brought into agreement with the central region of the curve at 0.64μ. This necessitated a slight change of the values at 0.65μ and at 0.66μ as found in the present investigation. For the blue end the published data of

[1] *Loc. cit.*

Hartman[1] are chosen as the best. The reasons for choosing these data for the red and blue ends are as follows:

1. They were obtained by a direct-comparison rather than by a flicker method, even though the size of the field of view was much smaller than that employed in the investigation of the middle of the spectrum described in the present paper.

2. They were obtained under favorable conditions as to brightness, and are probably more free from errors due to scattered light and slit-width corrections than other published data.

3. They are more definitely applicable in computing the transmitted luminosity of optical pyrometer screens, for which they will probably find their largest application, since they were obtained under the conditions which are found in optical pyrometry.

4. They will serve as well as any other values in computing integral luminosities or the mechanical equivalent of light, since large differences in the accepted visibility data in these extreme regions of the visible spectrum produce negligibly small errors.

Nela Research Laboratory
National Lamp Works of General Electric Company
Nela Park, Cleveland, Ohio
May 1918

[1] *Loc. cit.*

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

SIXTH PAPER: ON THE DETERMINATION OF THE DISTANCES OF GLOBULAR CLUSTERS

By HARLOW SHAPLEY

I. INTRODUCTION AND SYNOPSIS

The derivation of parallaxes for globular clusters will contribute to several problems of general astrophysical interest. Through the introduction of a linear scale we may expect to learn much of the actual dimensions of these stellar groups, and of the relation to distance from their centers, of the luminosity, mass, spectrum, and star density, and, eventually, of the internal motions. Since we may also be able to estimate at least a lower limit for the total mass involved in each group, the opportunity is promising for a contribution to the rather meager supply of observed facts for studies in stellar dynamics. Further, a knowledge of the distances of these widely distributed systems, and of the highly luminous variable stars for which parallaxes will be obtained in the course of the same research, will add to our conception of the dimensions of the visible stellar universe. These problems are of enough importance to justify a considerable effort in the study of the distances of globular clusters and their distribution in space. As direct measurement of cluster parallaxes is out of the question, the procedure must be by other methods, such as those involving proper motions and luminosity correlations.

The range in the absolute brightness of stars in many globular clusters is now known to exceed ten magnitudes, and we have as yet no reason to believe that the dispersion of luminosity in any of the typical globular clusters is not strictly comparable with the known dispersion in the general galactic system. Our ordinary investigations of cluster magnitudes, therefore, involve many stars of relatively great luminosity. Among these very brightest stars

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 151.

reddish Cepheid variables of long period frequently appear, while among the stars two or three magnitudes fainter is found the typical cluster-type variable—a Cepheid with a first-type spectrum and a period less than a day.

For some years we have known that the longer-period Cepheids in the galactic system are also giant stars, and a casual examination of their motions indicates a fairly small dispersion in actual luminosity. The possibility is at once suggested, therefore, of utilizing the motions of galactic Cepheids to obtain a mean value of the absolute magnitude of such stars, which, when compared with observed magnitudes of analogous objects in globular clusters, permits an estimate of the distances of the clusters themselves. One obvious advantage of operating with a group of giant stars is that in many of the most distant clusters only the stars of highest luminosity are within reach of our greatest telescopic power. It is also clearly of prime importance in problems of this nature, which involve faint stars and great distances, to have reliable systems of magnitudes established and to be able to ignore with safety the general scattering of light in space.

The methods and results of the investigation of cluster parallax will be discussed in Papers VI to XII, inclusive, of the series of studies of magnitudes in clusters. The present discussion begins with the derivation of a mean absolute magnitude for the isolated Cepheids whose motions are well determined, and then considers the remarkable relation between the period of variation and the total light-emission of such stars. Examples of this interesting correlation, which may perhaps be considered a fundamental law in Cepheid variation, are found among the variables in no less than six different stellar organizations as well as among the galactic Cepheids. Combined with the very homogeneous and accordant data derived from motions, it permits the deduction of absolute magnitudes, and hence of absolute distances, with a surprisingly small computed probable error. In fact, unless some intrinsic weakness in the procedure or some overlooked alternative is found, we may believe that the distances of the Cepheid variables, and of the extremely remote globular clusters in which Cepheid variables have been studied, can be determined with a percentage accuracy

rarely excelled and usually unequaled by direct measures of the nearest stars.

Now it happens that in many globular clusters no variable stars are as yet known; in others the Cepheids similar in period to those for which motions are known in the galactic system are of rare occurrence compared with the typical cluster-type Cepheid with period less than a day. We find, however, that the median magnitude of the typical cluster-type variable shows such an extremely small dispersion in a given cluster that a constant value can be assumed; and when we adopt the reasonable and important hypothesis that the equality of the total light-emission is universal for such variables, the corresponding absolute median magnitude can be accurately derived from the analysis of the motions, magnitudes, and periods of the longer-period Cepheids. The determination of the parallaxes of certain clusters then becomes a by-product of the study of their typical variable stars.

A further step correlates this median magnitude with the magnitude of the brightest stars in a cluster. A means is thus afforded of computing distances of all the clusters (whether or not they contain known variables) for which apparent photographic magnitudes of the brightest stars are measured. Finally, we find from the results derived by the processes outlined above that the parallax of a cluster is very definitely related to its angular diameter, and this yields a method of obtaining from already available photographs the distances of all globular clusters so far discovered.

In the second article (the seventh paper of the general series) the individual distances of globular clusters are derived; their highly significant distribution in space is also discussed, and some computations made of the linear dimensions, of the concentration of stars, and, provisionally, of the total masses in cluster systems.

The parallaxes and co-ordinates in space of all Cepheid variables, for which magnitudes and periods are known, appear in the eighth paper. The chief uncertainty in the results is due to the lack of accuracy and homogeneity in published values of magnitudes. For the isolated cluster-type variables accurate values of the period are not essential in deriving distances, as the dispersion in absolute magnitude is small for periods less than a day.

The next paper contains three notes on certain theoretical and observational aspects of Cepheid variation. An interpretation is offered for some of the results obtained in studying the distances of clusters and variables, the relation of color to period is examined observationally for cluster-type variables, and a composite color-curve is derived for more than a hundred variable stars.

An inquiry into some problems of the frequency of stellar luminosities appears in the tenth paper. The magnitudes of several thousand cluster stars have been measured for this discussion. Some light is thrown on the meaning of the median magnitude of variables, and the propriety of its use for parallax work, rather than the use of maximum or minimum, is established.

The eleventh paper treats briefly of the distances and distribution in space of various objects and classes of objects. The data derived from clusters and variables is supplemented by the results of other investigators for different types and groups of stars.

The last article, summarizing the facts and indications of the preceding papers, contains a preliminary attempt to discern the general plan of the sidereal system. The arrangement proposed has its principal justification in that it appears to be a simple interpretation of the new data and at the same time does not seem to be inconsistent with previous observational results bearing on the structure of the universe.

In the accumulation of the observational material for these studies much credit is due Miss Davis for the measurement of large numbers of stars in many difficult cluster fields. She has also assisted throughout in the reduction and discussion of the observations. Mrs. Shapley has collaborated in the treatment of most of the problems and, in particular, is responsible for large parts of the tenth and twelfth papers. Valuable assistance in the preparation of the papers for the press has been given by Miss Connor. Mr. Pease has freely permitted the use of his long-exposure cluster photographs. Data relative to parallaxes have been kindly furnished by Mr. van Maanen and relative to spectroscopic results by Mr. Adams. Special acknowledgment is due Mr. Seares for many valuable suggestions and criticisms and for painstaking editorial supervision of the whole series of cluster papers.

II. THE MEAN ABSOLUTE MAGNITUDE OF CEPHEID VARIABLES

Hertzsprung[1] and Russell[2] have computed the mean absolute magnitude of Cepheid variables from the proper motions given in the *Preliminary General Catalogue* of Boss, and the former has published his work in some detail. With a few corrections to the observational data the computations are now repeated and somewhat extended. The material is not extensive, and its sufficiency for this problem might well be questioned if it were not for the fortunate circumstances that the data are complete for each star and in most respects homogeneous; that no evidence of preferential motion is found; and that the peculiar motions of such stars, without exception, are small compared with their parallactic drifts.[3] Hertzsprung tabulates 13 stars, but two of them are not typical Cepheid variables and have been excluded;[4] no others have sufficiently accurate proper motions to be added to the list.[5]

Table I contains observational data relative to the group of 11 Cepheids. With the exception of Polaris, all are near the

[1] *Astronomische Nachrichten*, **196**, 201, 1913; see also *Zeitschrift für wissenschaftliche Photographie*, **5**, 94, 107, 1907.

[2] *Science*, N.S., **37**, 651, 1913.

[3] See n. 1, p. 103.

[4] Both the period and light-curve of κ Pavonis are subject to considerable perturbation according to Gould (*Uranometria Argentina*, p. 244, 1879), and Roberts also notes the star as an exception to ordinary Cepheid variation. The light-variation of *l* Carinae is peculiar; "an irregular and ill-defined secondary maximum has frequently been observed" (Roberts, *Astronomical Journal*, **21**, 89, 1901). Albrecht has just reported a variation in the spectrum of *l* Carinae from F8 to G5 (*Popular Astronomy*, **25**, 519, 1917), and it may be the star is not so abnormal as was believed when the computations for this paper were made.

The typical Cepheid characteristics of the eleven stars used are attested not only by their light-curves, but also in every case by spectroscopic study. We have no velocity-curves for *l* Carinae and κ Pavonis. The inclusion of both in the discussion would change the final result by less than its probable error. Including κ Pavonis would greatly decrease the certainty of the computed absolute magnitudes, though not altering them seriously, while the inclusion of *l* Carinae alone would make no appreciable difference in either the result or its probable error.

[5] The values given by Perrine in *Astrophysical Journal*, **41**, 308, 1915, for Y Ophiuchi and SZ Tauri are too uncertain for the present work. The proper motion of RR Lyrae, period 13.5 hours, is rightly excluded; see the eighth paper of this series, "The Luminosities and Distances of 139 Cepheid Variables," *Mt. Wilson Contr.*, No. 153, 1917.

TABLE I

Observational Material for Eleven Cepheids

Boss No.	Name	R.A. 1900	Decl. 1900	Galactic Longitude	Galactic Latitude	Median Magnitude	Spectrum	Period	τ	υ	λ	km/sec.
325..	α Ursae Minoris	$1^h 22^m.6$	+88° 46′	90°	+27°	2.12	F8	$3^d.9681$	+0″.016	+0″.041	60°	−15
637..	SU Cassiopeiae	2 43.0	+68 28	101	+ 9	5.9	A8–F5	1.9495	+ 8	+ 14	75	− 7
1629..	RT Aurigae	6 22.1	+30 34	151	+11	5.3	A7–G1	3.72806	− 4	+ 24	118	+22
1815..	ζ Geminorum	6 58.2	+20 43	164	+13	4.0	G	10.15382	− 2	+ 9	126	+ 7
4493..	X Sagittarii	17 41.3	−27 48	329	− 1	4.7	F1–G5	7.01188	− 3	+ 23	59	−14
4564..	W Sagittarii	17 58.6	−29 35	330	− 5	4.7	A8–G2	7.5946	+ 9	+ 12	61	−29
4632..	Y Sagittarii	18 15.5	−18 54	340	− 4	5.8	F4–G4	5.7734	+ 6	+ 12	50	+ 4
5071..	η Aquilae	19 47.4	+ 0 45	8	−14	4.05	A8–G5	7.176382	+ 1	+ 12	39	−14
5098..	S Sagittae	19 51.5	+16 22	23	− 7	5.8	F4–G3	8.381613	− 1	+ 4	30	−12
5370..	T Vulpeculae	20 47.2	+27 52	40	−11	5.8	A9–G1	4.435521	− 11	+ 10	36	− 1
5807..	δ Cephei.......	22 25.4	+57 54	73	+ 1	4.14	F0–G2	5.366386	+0.003	+0.012	52	−17

galactic plane; their distribution in galactic longitude is satisfactory. The median visual magnitudes differ in some instances from those used by Hertzsprung; the spectral types, taken mainly from *Contribution* No. 124,[1] are known to be variable for all but two of the stars. The proper motions have been reduced to the parallactic system of co-ordinates, υ being counted positive in the direction of the antapex. Without further reduction the parallactic motion is conspicuously evident, the unweighted algebraic means being

$$\left.\begin{array}{l}\tau=+0''.002\\ \upsilon=+0''.016\end{array}\right\} \qquad (1)$$

All values of υ are positive. The distances from the apex of the solar motion, λ, are very favorable for the weight of the solution.

Curves of velocity-variation have been determined for all of these stars, mainly at the Lick Observatory. Seven of the values of V, the observed radial velocity of the center of mass, are taken from Duncan's compilation in *Lick Observatory Bulletin*, No. 151. The value for α Ursae Minoris, computed by Miss Hobe, has been given by Campbell,[2] and those for X Sagittarii[3] and δ Cephei[4] were computed by Moore. The value for SU Cassiopeiae is obtained from an unpublished discussion by Adams and Shapley of the variations in light, velocity, and spectral type.[5]

Some of the steps in the reduction appear in Table II. To eliminate possible effects of the dispersion in distance the proper motions were reduced to the common apparent magnitude +5, which is very near the mean apparent median magnitude of the variables, +4.8. The weights of the individual values of the reduced parallactic motion, $\upsilon_5/\sin\lambda$, were determined in the manner suggested and used by Hertzsprung.[6] They depend on the distribution of velocities for this type of star, on the probable errors

[1] *Astrophysical Journal*, **44**, 273, 1916.

[2] *Lick Observatory Bulletin*, **6**, 19, 1910.

[3] *Ibid.*, **5**, 111, 1909.

[4] *Ibid.*, **7**, 153, 1913.

[5] *Mt. Wilson Contr.*, No. 145, 1917. *Note added to proof, April, 1918:* The source of the visual magnitude 6.23 is *Harvard Annals*, **50**, and not the *Preliminary General Catalogue*, as erroneously printed (*Astrophysical Journal*, **47**, 50, 1918).

[6] *Astronomische Nachrichten*, **196**, 203, 1913.

given by Boss for the individual proper motions, and on the distances from the solar apex. The weighted mean value and its probable error are

$$\frac{v_5}{\sin\lambda}=+0''.0161\pm0''.0016. \tag{2}$$

It is of interest to compare this value with (1). The corresponding value derived by Hertzsprung, if we reduce his result to the same apparent magnitude, is

$$\frac{v_5}{\sin\lambda}=+0''.0146\pm0''.0022$$

from which he derived for the mean parallax of a Cepheid of the fifth apparent magnitude

$$p_5=0''.0035\pm0''.00054,$$

and for the mean absolute magnitude, corresponding to the mean period of 6.6 days,

$$M=-2.3\pm0.35.$$

The larger probable error of his value of $v_5/\sin\lambda$ is attributable mostly to the inclusion of the highly discordant κ Pavonis.

TABLE II

SOLUTION FOR MEAN PARALLAX

Boss No.	Observed τ_s	Observed v_s	$\frac{v_s}{\sin\lambda}$	Relative Weight	Parallactic v_s	Residual v_s	V_0	π (M = −2.35)
325..	+0''.004	+0''.011	+0''.013	99	+0''.014	−0.003	− 4 km	0''.0128
637..	+0.012	+0.021	+0.022	48	+0.016	+0.005	− 2	22
1629..	−0.005	+0.028	+0.032	61	+0.014	+0.014	+12	30
1815..	−0.001	+0.006	+0.007	83	+0.013	−0.007	− 6	54
4493..	−0.003	+0.020	+0.023	80	+0.014	+0.006	− 4	39
4564..	+0.008	+0.011	+0.013	69	+0.014	−0.003	−18	39
4632..	+0.009	+0.017	+0.022	12	+0.012	+0.005	+18	24
5071..	+0.001	+0.008	+0.013	48	+0.010	−0.002	+ 2	52
5098..	−0.001	+0.006	+0.012	16	+0.008	−0.002	+ 6	24
5370..	−0.016	+0.014	+0.024	11	+0.009	+0.005	+16	24
5807..	+0.002	+0.008	+0.010	79	+0.013	−0.005	− 4	0.0050

With the value of the sun's velocity adopted by Hertzsprung, 20 km/sec., equation (2) gives $p_5=0''.00386$ and $M=-2.06$; but later

determinations from radial motions point to a higher velocity of the sun. We have the following recent values, based on a greater proportion of giant stars than were included in the early determinations and therefore probably more reliable because of smaller peculiar velocities:

		km/sec.	Mean Error
Charlier,[1]	156 B-type stars (mag.<5.0), $V_\odot$	=21.26	
Gyllenberg,[2]	284 B-type stars	=22.06	±0.91
Gyllenberg,	291 A-type stars	=19.77	±1.45
Strömberg,[3]	1400 F, G, K, M, giant stars	=21.48	±1.02

Adopting $V_\odot = 21.5$ km/sec.,

$$\frac{v}{p \sin \lambda} = 4.535$$

$$p_5 = 0\rlap{.}''00354 \pm 0\rlap{.}''00035 \tag{3}$$

and the absolute magnitude, corresponding to the mean period of 5.96 days, is

$$M = -2.26 \pm 0.22.$$

Computing for each star that part of the v-component due to parallactic motion, $0\rlap{.}''0161 \sin \lambda$, and subtracting these values in the sixth column of Table II from the observed values of v_5 in the third column, we obtain in the seventh column values for the peculiar velocity parallel to the direction of the sun's motion. The values of τ_5 and of "residual v_5" are in satisfactory agreement.[4] Their mean values, without regard to sign, are

$$\tau_5 = 0\rlap{.}''0056 \pm 0\rlap{.}''0010.$$

$$\text{Residual } v_5 = 0.0052 \pm 0.0007.$$

[1] *Meddelande från Lunds Astronomiska Observatorium*, Serie II, No. 14, 1916.

[2] *Ibid.*, No. 13, 1915.

[3] *Mt. Wilson Contr.*, No. 144, 1917.

[4] A considerable part of the agreement is a necessary consequence of the magnitude of the observational errors relative to those of the proper motions. The average probable error for the annual motion in one direction is about $\pm 0\rlap{.}''004$. As the peculiar motions are not greatly in excess of the errors of observation, possibly less weight should have been given to (6) in obtaining the final value (7). The difference between (3) and (7), however, is less than half the probable error of either.

Combining all twenty-two values, with half weight for the residual v_5=components because one constant has been derived from them, we obtain the mean value for the peculiar proper motion in one direction

$$\mu_5' = 0\overset{''}{.}0055 \pm 0\overset{''}{.}00064, \tag{4}$$

a value about one-third that of the derived parallactic motion.

The observed radial velocities corrected for the sun's motion[1] are given in the eighth column of Table II. Their arithmetical mean value is

$$V_0 = 8.35 \pm 1.29 \text{ km/sec.} \tag{5}$$

and we derive from (4) and (5) an independent value of the mean parallax

$$p_5 = \frac{4.74\,\mu_5'}{V_0} = 0\overset{''}{.}0031 \pm 0\overset{''}{.}0005. \tag{6}$$

Probably the very close agreement of (3) and (6) is partly chance. If in the place of (5) we use a value based upon all known radial velocities of Cepheids, we add to the eleven values in Table II nine others, most of which are only rough estimates,[2] and obtain $V_0 = 9.58$ km/sec., $p_5 = 0\overset{''}{.}0027$.

Giving double weight to (3), or, what amounts to the same, combining (3) and (6) with regard to their probable errors, we obtain the final values:

$$\left.\begin{aligned} p_5 &= 0\overset{''}{.}0034 \pm 0\overset{''}{.}00030 \\ M &= -2.35 \pm 0.19 \end{aligned}\right\} \tag{7}$$

The probable errors of the foregoing mean values include the errors in the observed proper motions and radial velocities and some uncertainties of reduction; but they do not include the errors

[1] The value 21 km/sec. was used for the sun's velocity in this computation, but no difference in the mean value would result from using 21.5 km/sec.

[2] The additional stars are SU Cygni, Y Ophiuchi, SZ Tauri, T Monocerotis, and five southern Cepheids for which very approximate velocities have been estimated recently by Paddock from a few plates of each (*Lick Observatory Bulletins*, **9**, 68, 1917). The value for T Monocerotis is a little more than a guess (*Astrophysical Journal*, **23**, 266, 1906). The velocity-curve of SZ Tauri is by Haynes (*Lick Observatory Bulletins*, **8**, 85, 1914), and for the others Duncan's table furnishes information.

(relatively much less important in the derivation of M) in the apparent magnitudes and the velocity of the sun, nor the uncertainty arising from any systematic motion or drift of the Cepheids as a group.

From the value (7) and the apparent median magnitudes in the seventh column of Table I, the individual parallaxes have been derived and entered in the last column of Table II. The probable error of each value depends only on that of M and the uncertainty of the corresponding apparent magnitude; its average is about $\pm0''.0007$, or 15 per cent of the tabulated parallaxes. Better values of these parallaxes are derived later.[1]

TABLE III

Boss Number	Observer	Rel. π and P.E.	Absolute π	π from Parallactic Motion and Period
325	Flint *et al.*	$\pm0''.018$	$+0''.028$	$+0''.016$
637	van Maanen	$+0.008\pm0.003$	$+0.010$	$+0.004$
1815	Abetti, Miller	$+0.019\pm0.009$	$+0.025$	$+0.004$
5071	Mitchell	$+0.001\pm0.009$	$+0.006$	$+0.005$
____(X Cygni) . .	Miller	0.000 ± 0.008	$+0.006$	$+0.001$
5532	Lee and Joy	-0.016 ± 0.014	-0.011	$+0.018$

Direct measures of the parallaxes of four of the 11 Cepheids here considered are available for comparison with the results of the present investigation. The data are shown in the first four lines of Table III. Flint's parallax (absolute) of Polaris, $+0''.008\pm0''.016$, is combined with six of the more recent determinations listed by Kapteyn in *Groningen Publications*, No. 24. The values of the last column are taken from a later paper of this series. The absolute values are systematically larger than those based upon the parallactic motion, the mean difference being $+0''.010$ for the first four stars, which corresponds to a difference of approximately two magnitudes in the absolute brightness. This discrepancy is large and raises a question as to possible sources of error.

We note, however, that the inclusion of X Cygni and the short-period variable β Cephei (Boss 5532)[2] reduces the difference in the

[1] *Mt. Wilson Contr.*, No. 153, the eighth paper of this series.

[2] Their parallaxes are determined later by the method used to calculate the final values for the 11 Cepheids, although they could not be included in the original group.

mean values to $+0''.003$; and, further, according to an unpublished investigation by van Maanen, there is some evidence that the directly measured parallaxes require systematic corrections sufficient to bring the mean value below that derived from the parallactic motion by $0''.001$. These circumstances illustrate the uncertainty of any conclusion based upon a small number of directly measured parallaxes whose values are as minute as those of the Cepheid variables appear to be.

On the other hand, the smallness of the proper motions makes the calculated parallactic motion sensitive to any systematic errors in the observed motions, provided the errors are so large or so distributed that the mean value of p_s is affected. An attempt to determine directly from the observations a systematic correction to the proper motions is not expedient, however, as the number of stars is small and they are widely scattered over the sky. The reduced individual motions should reveal any conspicuous error.

If we assume a systematic correction to the proper motions of the amount used by Kapteyn (*Contributions*, Nos. 82 and 147) for some of the B-type stars in the southern hemisphere, and suppose that its effect is not obliterated in the mean result, the deduced absolute brightness might be changed by about one-tenth of a magnitude, an amount clearly insufficient to question the general accuracy of the present result.

In the introduction to the *Preliminary General Catalogue* Boss has suggested as provisional systematic corrections to his proper motions:

$$d\mu_\alpha = +0^s.00021 - 0^s.00015 \sin\alpha \tan\delta$$
$$d\mu_\delta = -0''.0023 \cos\alpha$$

These corrections have been applied to the proper motions of the Cepheids and revised values computed for the τ- and υ-components. In Table IV the probable errors of the proper motions, as given by Boss, should be compared with the proposed systematic corrections. (For the first star, Polaris, a systematic correction of this general nature is of course not appropriate.)

In nearly every case the probable error exceeds the systematic correction, and in the average it is about three times as large. The

revised values of τ and υ should be compared with the analogous values in Table I. The average correction to υ is just one-half the probable error of the proper motion in one direction, and the systematic correction to the parallactic motion is also about one-half of its probable error. The corresponding correction to the adopted absolute magnitude would be less than +0.05, an amount safely negligible. We conclude, therefore, that unless the systematic errors are assumed to be of a wholly different order of magnitude from those derived by Kapteyn and Boss the mean parallax of these Cepheids is essentially correct.

TABLE IV

Boss No.	P.E. of μ_α	P.E. of μ_δ	$d\mu_\alpha$	$d\mu_\delta$	Revised		$\Delta\upsilon$
					τ	υ	
325....	±0ˢ.00343	±0″.0008	(−0ˢ.00225)	(−0″.0022)	+0″.015	+0″.041	0″.000
637....	88	40	− 4	− 17	+ 7	+ 16	+ 2
1629....	37	35	+ 12	+ 2	− 2	+ 23	− 1
1815....	11	17	+ 16	+ 6	0	+ 7	− 2
4493....	23	30	+ 13	+ 1	− 2	+ 22	− 1
4564....	35	44	+ 12	+ 0	+ 11	+ 12	0
4632....	74	80	+ 16	− 2	+ 8	+ 12	0
5071....	20	29	+ 21	− 10	+ 3	+ 15	+ 3
5098....	31	40	+ 25	− 11	0	+ 8	+ 4
5370....	59	63	+ 27	− 15	− 11	+ 13	+ 3
5807....	±0.00020	±0.0017	+0.00031	−0.0021	0.000	+0.014	+0.002

There remains, however, the possibility of systematic error in the mean distance due to a preferential drift of the Cepheids as a class—a drift which, in order to escape ready detection by means of the peculiar motions, must be either very small or nearly in the direction of the solar motion. Suppose, for example, that the mean absolute magnitude of the eleven Cepheids were zero rather than −2.35. This corresponds approximately to the extreme systematic difference indicated by the directly measured parallaxes. Then the mean parallactic motion would be 0″.05, that is, three times the observed value (which is determined with a computed error of only 10 per cent). To harmonize this assumption with the observed parallactic motion we must assume an annual preferential drift of 0″.03 in a direction deviating but a few degrees from the solar apex. Such a drift should reveal itself in the radial

velocities, which show no effect of the required order of magnitude. As a matter of fact, the mean component of these velocities (corrected for solar motion) in the direction of the sun's apex is $+3.3$ km/sec. with a probable error of ± 3.5. The corresponding effect on the mean parallactic motion would be of the order of $0\overset{''}{.}003$, or one-tenth that required; and the uncertainty, arising from this source, in the mean parallax given in (7) is perhaps of the order of $0\overset{''}{.}0006$.

TABLE V

Boss No.	p'_5	Probable Error	M	Period	π
325........	$0\overset{''}{.}0029$	$\pm 0\overset{''}{.}0009$	-2.7	$3\overset{d}{.}97$	$0\overset{''}{.}0110$
637........	49	13	-1.6	1.95	32
1629........	71	11	-0.7	3.73	63
1815........	15	10	-4.1	10.15	24
4493........	51	10	-1.5	7.01	58
4564........	29	11	-2.7	7.59	33
4632........	49	26	-1.6	5.77	33
5071........	29	13	-2.7	7.18	45
5098........	26	22	-2.9	8.38	18
5370........	53	27	-1.4	4.44	36
5807........	0.0022	± 0.0010	-3.3	5.37	0.0033

A general solution for the preferential motion, based on all the Cepheids for which radial velocities are available,[1] gives the following values for the velocity of the hypothetical drift, its rectangular components, and the position of its apex:

$$V = 6.4 \pm 3.0 \text{ km/sec.}$$

$$X = +0.2 \pm 6.2 \qquad Y = -1.8 \pm 2.6 \qquad Z = +6.1 \pm 3.0$$

$$A = 275^\circ \pm 195^\circ \qquad D = +74^\circ \pm 131^\circ$$

The relative magnitudes of the probable errors show that as far as this material goes there is no appreciable motion of the Cepheids as a class.

Referring again to the data of Table II, we assume, for the purpose of computation, that the υ-component for every star in the foregoing tables is wholly parallactic. Then the parallax of each variable, reduced to apparent magnitude $+5$, is $p'_5 = 0.22\ \upsilon_5/\sin\lambda$, and the corresponding absolute magnitudes are as given in Table V. Proceeding as above, where a constant value of M was used, new

[1] See n. 2, p. 98.

values of π are obtained. Their fairly close agreement with the results of the last column of Table II further emphasizes the relative smallness of the neglected peculiar motion.[1]

The chief interest of Table V, however, lies in the comparison of the absolute magnitude with the period of variation. A marked correlation is at once evident,[2] and if we combine the stars in order of period into small groups so as to eliminate some of the effect of the neglected peculiar motion in υ and some of the errors of observation, we find

Number of Stars	Absolute Magnitude	Period
1	−4.1	10.15 days
3	−2.8	7.72
3	−2.1	6.05
3	−1.6	4.05
1	−1.6	1.95

The probable error of the mean absolute magnitude would be further reduced by allowing for this correlation between luminosity and period.[3]

III. THE RELATION OF PERIOD TO LUMINOSITY

From the plot[4] of the numbers in the fourth and fifth columns of Table V a new absolute magnitude is obtained for each star, and

[1] This agreement further suggests that the number of stars is sufficient to give a dependable value of the mean parallax. We may, in fact, group the stars in threes, either at random, or in the order of any characteristic (except period), and the separate means will almost invariably give a value differing by less than half a magnitude from the mean value for the eleven stars.

[2] This might be interpreted as a peculiar distribution of the υ-components depending on the length of the period, but further work on the relation of period to luminosity completely disposes of this unlikely alternative.

[3] The correction, however, is very small because the principal source of the probable error remains in the dispersion of the residual υ_S-components. Since $M=f(P)$, we observe that the reduced parallactic motion, $\upsilon_S/\sin\lambda$, is also a function of the period, and that for the extremes of period the residual υ_S values really contain some portion (positive or negative) of the parallactic motion. A new solution, which involved the reduction of the observed proper motions to apparent magnitudes depending on the period rather than to the common apparent magnitude +5, naturally gave no appreciable difference in the mean parallax and absolute magnitude, and reduced the probable error of the latter by less than 0.02.

[4] The deviations from the smooth curve correspond in general to the residual υ_S-components, providing we grant that M is uniquely defined by P.

these smoothed values are plotted against the logarithm of the period as large circles in Fig. 1. From a final curve, based on

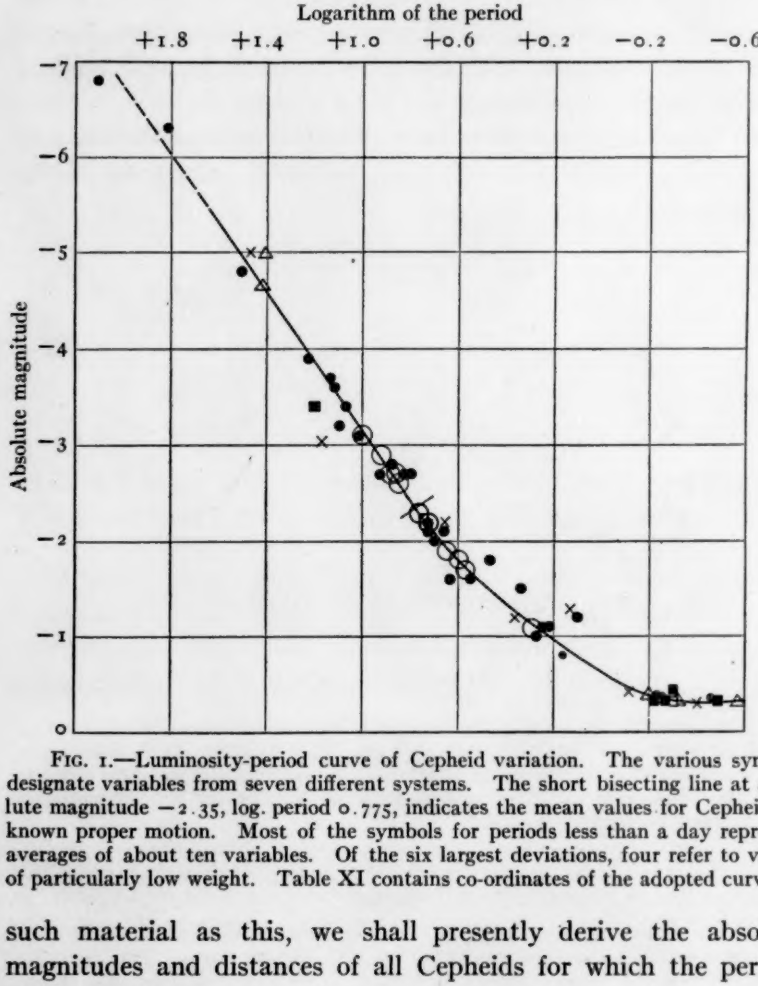

FIG. 1.—Luminosity-period curve of Cepheid variation. The various symbols designate variables from seven different systems. The short bisecting line at absolute magnitude −2.35, log. period 0.775, indicates the mean values for Cepheids of known proper motion. Most of the symbols for periods less than a day represent averages of about ten variables. Of the six largest deviations, four refer to values of particularly low weight. Table XI contains co-ordinates of the adopted curve.

such material as this, we shall presently derive the absolute magnitudes and distances of all Cepheids for which the periods are known.[1]

[1] The eighth paper of this series contains the results for the individual stars (*Mt. Wilson Contr.*, No. 153). Without correcting for the progression of color with period, and assuming a linear relation between period and luminosity, absolute magnitudes have already been computed for two-thirds of the long-period Cepheids by Hertzsprung (*Astronomische Nachrichten*, **196**, 205, 1913), and by Russell for a paper by Russell and Shapley (*Astrophysical Journal*, **40**, 417, 1914). The individual results were not published.

Some years ago Miss Leavitt found a similar relation between the apparent photographic brightness and the length of period for the Cepheid variable stars in the Small Magellanic Cloud.[1] In order to support the result derived above for isolated Cepheids and to obtain a definitive luminosity-period curve, we may reduce her results to the absolute visual system of the present work. From certain globular clusters information that is still more valuable may be obtained through the correlation of the luminosities of the long- and short-period Cepheids. The various sources will be taken up separately.

1. *Small Magellanic Cloud.*—The magnitudes given by Miss Leavitt for stars in the Magellanic clouds are based upon estimates on ordinary photographic plates and are referred to a provisional scale and zero-point. The uncertainty of the zero-point is of no importance for our immediate purpose. As the magnitude scale is probably nearly correct, we shall adopt it as it stands, giving diminished weight to the brightest and faintest stars, and transforming the median brightness of the variables from photographic to visual apparent magnitudes.

The reduction to the visual system will be very small for the short-period variables, but probably as much as two magnitudes for some of the red stars with periods longer than ten days. In the absence of direct color or spectral determinations the change to visual magnitudes must be made on the basis of length of period, using the data already collected for an earlier paper.[2] With a few modifications based on recent spectral classifications,[3] the material is given again in Table VI, the last two columns of which are plotted in Fig. 2. The curve as drawn in the figure has been used for all color corrections; but for periods greater than one day a linear formula

$$\text{Color-index} = -0.55 + 1.5 \log P$$

represents satisfactorily the change of color with period. The probable interpretation of the curve is discussed in a later article.[4]

[1] *Harvard Circular*, No. 173, 1912; *ibid.*, No. 79, 1904; *Harvard Annals*, **60**, 106, 1908.

[2] *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[3] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

[4] "Three Notes on Cepheid Variation," the ninth paper of this series.

Of the 969 known variables[1] in the Small Magellanic Cloud the periods of 25 have been determined. The designation, logarithm of

TABLE VI

PERIOD AND SPECTRAL TYPE

Limits of Spectrum	Mean Spectrum	Number of Stars	Color-Index	Log. of Mean Period
Bo to A9...	A4	15	+0.15	−0.26
Fo to F9...	F5	27	+0.6	+0.78
Go to G9...	G5	31	+1.0	+1.04
Ko to K9...	K5	9	+1.4	+1.26
M+....	Ma	3	+1.6	+1.52

the period, median[2] photographic magnitude, reduction for color-index by means of the curve in Fig. 2, and the adopted visual magnitude are given for each star in successive columns of Table VII.

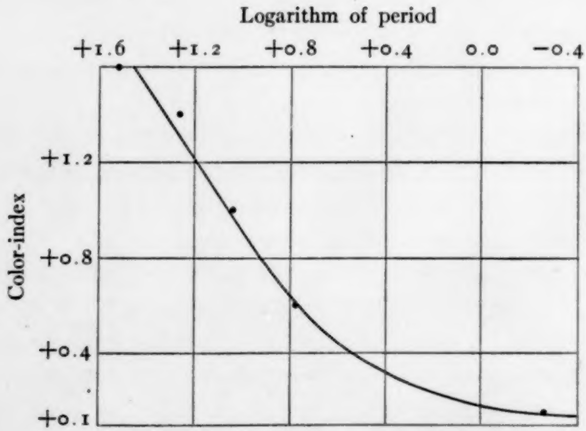

FIG. 2.—Change of median color with period for Cepheid variables

The visual magnitudes of the 25 variables were reduced as follows to the absolute system represented in Fig. 1. The provisional luminosity-period curve, based on the isolated variables as described in an earlier paragraph, covers only a part of the interval for which periods have been determined in the Small

[1] *Harvard Annals*, **60**, No. IV, 1908.

[2] The mean of the values for maximum and minimum.

Magellanic Cloud. To reduce to the absolute scale, values of the absolute magnitude corresponding to the observed periods of the second column have been read from the provisional curve for the 13 variables with $\log P$ between 0.27 and 1.00. The results are in the sixth column of Table VII; in the seventh are the

TABLE VII

CEPHEID VARIABLES IN SMALL MAGELLANIC CLOUD

Harvard Variable	Log. Per.	Median Pg. Mag. (Leavitt)	Color-Index from Curve	Median Vis. Mag.	Absolute Mag. from Curve	Difference	Provisionally Adopted Abs. Mag.
1505....	0.098	15.4	+0.2	15.2	....	....	−1.2
1436....	0.221	15.6	0.3	15.3	....	....	1.1
1446....	0.246	15.6	0.3	15.3	....	....	1.1
1506....	0.273	15.7	0.3	15.4	−1.1	−16.5	1.0
1413....	0.337	15.2	0.3	14.9	1.3	16.2	1.5
1460....	0.464	15.0	0.4	14.6	1.5	16.1	1.8
1422....	0.545	15.3	0.4	14.8	1.7	16.5	1.6
842....	0.632	15.3	0.5	14.8	1.9	16.7	1.6
1425....	0.658	14.8	0.5	14.3	2.0	16.3	2.1
1742....	0.698	14.9	0.5	14.4	2.1	16.5	2.0
1646....	0.726	14.9	0.6	14.3	2.2	16.5	2.1
1649....	0.727	14.8	0.6	14.2	2.2	16.4	2.2
1492....	0.799	14.3	0.6	13.7	2.3	16.0	2.7
1400....	0.823	14.4	0.7	13.7	2.4	16.1	2.7
1355....	0.874	14.4	0.8	13.6	2.7	16.3	2.8
1374....	0.924	14.5	0.8	13.7	−2.8	−16.5	2.7
818....	1.015	14.2	0.9	13.3			3.1
1610....	1.066	14.0	1.0	13.0	Mean diff.	−16.4	3.4
1365....	1.094	14.3	1.1	13.2			3.2
1351....	1.116	13.9	1.1	12.8			3.6
827....	1.129	13.8	1.1	12.7			3.7
822....	1.224	13.8	1.3	12.5			3.9
823....	1.504	13.2	1.6	11.6			4.8
824....	1.818	12.1	2.0	10.1			6.3
821....	2.104	11.6	+2.0	9.6			−6.8

differences between the magnitudes from the curve and the apparent magnitudes. The mean difference, −16.4, is the reduction constant, which, applied to all apparent median magnitudes, gives in the last column the corresponding absolute magnitudes. In making this transformation, the purpose of which is to relate *change* of period with *change* in absolute visual magnitude, we make no assumption regarding the actual luminosity of Cepheid variables in the Magellanic clouds; but as soon as we use the reduction constant as a measure of distance we assume, of course, that variables

of a given period are of comparable luminosity, whether they are in the general galactic system or in separate stellar organizations, such as globular clusters and the Magellanic clouds.

The absolute magnitudes of the last column of Table VII, plotted in Fig. 1 as dots, show that the same relation holds for the Cepheids in the Small Magellanic Cloud and in the galactic system, and that an improved and extended luminosity-period curve may be based upon the combined data. The relation appears so definite that the prediction of the length of period on the basis of magnitude estimates should be possible for most of the 944 other variables in the Small Magellanic Cloud.[1] Two or three hundred of them are fainter than any for which the period has been determined. It is very probable that they are cluster-type variables with periods of the order of twelve hours. With no correction either for color or for divergence of magnitude scale their median magnitudes on the absolute system are about -0.3, agreeing almost exactly with the mean value determined below for cluster-type variables in globular clusters.

It is important to note further that magnitude 16.0 in Miss Leavitt's provisional system marks an abrupt and definite fainter limit to the median brightness of the variables in the Small Magellanic Cloud. The plates, which were made with the 24-inch Bruce telescope with exposures varying from two to five hours, are sufficient to test this matter, for minima are observed as faint as 17.0, but, with one possible exception, no maximum is recorded fainter than 16.0. A similarly definite fainter limit to the interval of magnitude throughout which Cepheids occur has been observed in globular clusters, particularly in ω Centauri, and Messier 3, 5,

[1] Without doubt nearly all variables in both clouds belong to the Cepheid class. In the Small Cloud, however, four variables have an observed range of at least three magnitudes, and are probably long-period variables rather than Cepheids. Possibly a few others have larger ranges of variation than shown by the plates examined by Miss Leavitt and are also long-period variables. It seems to be a definite observational fact that no star that otherwise has typical Cepheid characteristics is known to have a range in excess of two magnitudes. The value, photographically, falls usually between 0.8 and 1.5, regardless of the length of period. Hence the appropriateness of Eddington's search for the physical reason of an upper limit to the amplitude of a pulsation in a gaseous star ("The Pulsation Theory of Cepheid Variables," *Observatory*, **40**, 290, 1917).

and 13. The concurrent progression toward a definite limit of luminosity, spectrum, and period suggests that, in the evolutionary sequence of stars, Cepheid variation is abruptly limited at or near the blue end of the giant series because of the changing physical conditions in the interior of the gaseous masses.

2. *ω Centauri.*—Of 132 variables observed by Professor Bailey in the southern cluster ω Centauri,[1] periods were determined for 93, of which 3 are long-period Cepheids. From the observations and notes it has been possible to derive approximate results for two others with periods in excess of a day. In Table VIII the correction to visual magnitude and the reduction to the absolute system follow

TABLE VIII

CEPHEID VARIABLES IN ω CENTAURI

Designation	Log. Per.	Median Pg. Mag. (Bailey)	Visual Mag.	Mag. from Curve	Difference	Provisional Abs. Mag.
1..........	1.47	10.45	8.80	−4.70	−13.50	−5.00
29..........	1.17	11.94	10.76	−3.60	−14.43	−3.04
48..........	0.66	12.08	11.60	−2.00	−13.60	−2.20
60..........	0.13	12.74	12.52	−1.01	−13.51	−1.28
61..........	0.36	12.92	12.60	−1.35	−13.95	−1.20
a(37).......	−0.23	13.55	13.40			−0.40
b(19).......	−0.12	13.54	13.37	Mean diff.	−13.80	−0.43
c(34).......	−0.40	13.61	13.49			−0.31

the same plan as that for the Small Magellanic Cloud. The magnitudes of the fifth column were read from the improved luminosity-period curve, but since at this point in the discussion the curve has not yet been extended to periods less than a day, the three groups of cluster-type variables were not used in the reduction. The light-curve of No. 29 is hardly typical of Cepheids, and its magnitude also appears somewhat discordant. In obtaining the reduction constant, −13.80, half weight was given to the approximate magnitudes for Nos. 48 and 61. The numbers in parentheses in the first column refer to the total number of stars in each subgroup. Plotting as crosses in Fig. 1 the values of the second and last columns, the luminosity-period curve is further improved for the ordinary Cepheids and is extended to the cluster-type variables.

[1] *Harvard Annals*, **38**, 1902.

3. *Messier 5.*—Professor Bailey has recently determined the periods and light-curves of about 70 variables in Messier 5.[1] Of the three with periods longer than a day, two are certainly Cepheids. The third, No. 50 of Bailey's list, with a period of 106 days, is one component of a close bright double. Its observed magnitude is certainly diminished by the Eberhard effect, possibly to a great extent. Professor Barnard has observed visually all three of these stars.[2] In a recent letter he states that the measures of No. 50 do not give the rapid rise to maximum light that is typical of Cepheid variation. If the star were included without correction for the Eberhard effect, it would deviate more than three magnitudes from the curve.

TABLE IX

CEPHEID VARIABLES IN MESSIER 5

Designation	Log. Per.	Median Pg. Mag. (Bailey)	Visual Mag.	Mag. from Curve*	Difference	Provisional Abs. Mag.
42	1.41	11.72	10.20	−4.55	−14.75	−4.98
50	2.03	13.60	11.6	....	....	−3.6
84	1.42	12.08	10.53	−4.57	−15.10	−4.65
Double Max. (8)	−0.57	14.98	14.84	....	....	−0.34
Group 1	−0.32	14.98	14.85	−0.33	−15.18	−0.33
Group 2	−0.26	14.98	14.83	−0.39	−15.22	−0.35
Group 3	−0.20	14.96	14.80	−0.45	−15.25	−0.38
				Mean diff.	−15.18	

* It should be noted that the luminosity-period curve is slightly corrected after each accretion of data so that the magnitudes in the fifth column are not those derivable from the final curve.

A group of 8 variables with double maxima or peculiarly short periods is found among the cluster-type variables of Messier 5.[3] That they are single stars with an average period of six and one-half hours seems the most probable hypothesis; they are accordingly used to extend the luminosity-period curve and to show that no appreciable decrease in luminosity occurs as the periods become less than twelve hours.

Bailey has collected into three equal groups, in order of length of period, the thirty typical cluster-type variables with most definite light-curves. Each group is given weight 5 in determining the reduction constant. The material is discussed in Table IX

[1] *Harvard Annals*, **78**, Part 2, 1917.

[2] *Astronomische Nachrichten*, **147**, 243, 1898.

[3] *Harvard Circular*, No. 193, 1916.

and, omitting No. 50, the results are plotted in Fig. 1 as triangles.

4. *Messier 3.*—Bailey's monograph on Messier 3 contains the light-curves of 110 stars, none of which has a period exceeding a day.[1] In his catalogue the only bright star that is certainly variable is one that appears to be irregular. A bright Cepheid, however, was found near the center of the cluster by Barnard, who has determined the period and published a light-curve.[2] The median photo-visual magnitude of this star has been determined from plates made at Mount Wilson and measured by Miss Davis and the writer. Although the light-curve derived by Barnard is more nearly symmetrical than is usual for Cepheid variables, the color and the change of color between maximum and minimum, which is typical of this kind of variable, is clearly indicated by the Mount Wilson measures. A small correction to the final magnitude for Eberhard effect would be appropriate, because of the star's situation in the densest part of the cluster, and would probably eliminate its deviation from the curve in Fig. 1.

One variable of Bailey's list, No. 37, has been specially studied at Harvard[3] and on a series of Mount Wilson plates.[4] Its period is about one-half that of the typical variable of Messier 3, resembling in this respect, as well as in the shape of the light-curve, the eight

[1] *Harvard Annals*, **78**, Part 1, 1913.

[2] *Astronomische Nachrichten*, **172**, 345, 1906. Barnard suspected another bright star of variation, No. 19, in his nomenclature. Referred to his comparison star No. 8 no variation is apparent on the Mount Wilson plates so far examined. The data for photographic magnitudes are as follows:

Plate	Date	19−8	Plate	Date	19−8
2372	1915, April 16	$0^{M}.00$	3679	1917, March 28	$+0^{M}.06$
2463	June 7	$+0.10$	3680	28	$+0.04$
2506	July 6	$+0.04$			

These stars are among the very brightest in the cluster, and, as might have been inferred from previous investigations, they are red. The color-index of No. 19 is approximately $+1.8$ magnitudes, and of No. 8, more than two magnitudes, according to measures on five photographic and three photo-visual plates. There is some possibility of Eberhard effect as both stars are near the center.

[3] *Harvard Circular*, No. 193, 1917; *Harvard Annals*, **78**, Part 2, 1913.

[4] *Publications of the Astronomical Society of the Pacific*, **29**, 110, 1917.

stars of Messier 5 that are designated "Double Max." in Table IX. It is entered singly in Table X. The 54 typical cluster-type variables in Messier 3 for which Bailey considered the results most certain are combined, in order of period, into three equal groups, each of which is given weight 10 in deriving the reduction constant. The photo-visual magnitudes are referred to the Mount Wilson system, a series of polar comparison plates being used to obtain the appropriate correction to Bailey's photographic magnitudes. Black squares in the diagram designate the results for Messier 3.

5. *Messier 13.*—Two Cepheid variables in Messier 13 were discovered by Bailey and Barnard and studied by the latter.[1] Of five others found by the writer among the fainter stars[2] four appear

TABLE X

CEPHEID VARIABLES IN MESSIER 3

Designation	Log. Per.	Median Pv. Mag.	Mag. from Curve	Difference	Provisional Abs. Mag.
Barnard 7...	1.20	12.3	−3.7	−16.0	−3.4
Bailey 37....	−0.49	15.40	−0.35	−15.75	−0.33
Group 1.....	−0.30	15.29	−0.34	−15.63	−0.44
Group 2.....	−0.27	15.39	−0.36	−15.75	−0.34
Group 3.....	−0.22	15.39	−0.38	−15.77	−0.34
			Mean diff......	−15.73	

to have periods less than a day. Approximate median photo-visual magnitudes for these four and the earlier two have been obtained from Mount Wilson photographs; but without a complete study of all the light-curves the results have low weight. Carrying through the usual reductions, however, and combining the provisional results into two groups, we obtain the values indicated by open squares in Fig. 1.

6. *Other sources.*—Further numerical results bearing on the luminosity-period relation are not now available, but certain sources may be cited from which quantitative results will come in time and from some of which even now we may infer confirmation of the interdependence of period and luminosity.

[1] Bibliography and discussion in *Mt. Wilson Contr.*, No. 116, p. 78, 1915.

[2] *Ibid.*, pp. 79 ff.

The median photographic magnitudes of the 808 variables discovered by Miss Leavitt in the Large Magellanic Cloud[1] range from 10.5 to 15.8; there is but one fainter exception, which, by showing that fainter stars are easily visible on the plates, proves the rule that Cepheid variation affects only stars brighter than a certain limit. If we should assume the same color correction as used for the Small Magellanic Cloud, the interval of magnitude in which the variables occur is much the same. There can be little doubt that these are Cepheid variables (the ranges of all but two or three are less than two magnitudes[2]), among which the luminosity-period law holds as elsewhere. Good values of the distance and extent of the star cloud will be given eventually through determinations of the magnitudes and periods of these variable stars.

The 50 variable stars in Messier 15 are being studied by Professor Bailey on a series of 75 Harvard and Mount Wilson plates. The periods so far derived, with one exception, are less than a day. The exceptional star has a period of 1.44 days, a provisional value kindly communicated by Professor Bailey; its median magnitude is about 0.43 brighter than the median magnitude of the other variables, according to measures on a few Mount Wilson plates. The results are preliminary and receive no weight in obtaining the luminosity-period curve, but because of the close agreement with results for other clusters, the following data, derived in the usual manner, are plotted in Fig. 1 as small circles containing crosses:

Variable No. 1, Median Mag. (Absolute) $= -0.82$, Log. Per. $= +0.16$
Variable No. 13, Median Mag. (Absolute) $= -0.40$, Log. Per. $= -0.24$
Variable No. 11, Median Mag. (Absolute) $= -0.37$, Log. Per. $= -0.46$

Nos. 13 and 11 are typical of the two subclasses of variables in Messier 15; the median photographic magnitudes, Bailey finds, are in all cases about 15.7.[3]

Variable stars have now been discovered in 26 globular clusters,[4] and from Mount Wilson plates and from reproductions in *Harvard*

[1] *Harvard Annals*, **60**, Part IV, 1908; *Harvard Circular*, No. 82, 1904.

[2] See n. 1, p. 108.

[3] *Popular Astronomy*, **25**, 520, 1917.

[4] To the list in *Harvard Annals*, **38**, p. 2, several have been added through recent discoveries by Miss Davis (*Publications of the Astronomical Society of the Pacific*, **29**, 210. 260, 1917).

Annals, **38**, it is possible to make a preliminary comparison of the relative magnitudes in each system. Thus we find that there are at least 15 clusters, in addition to those examined in some detail in the foregoing discussion, in which some or all of the variables are among the very brightest stars. Clusters which show also a considerable diversity in the magnitude of variables, and which are therefore of importance for the luminosity-period curve, include N.G.C. 104, 362, 1904, 6266, 6397, and 6626.

The final luminosity-period curve, as drawn in Fig. 1 and given numerically in Table XI, is based upon more than 230 stars, and, except for zero-point uncertainty, is probably correct within one or

TABLE XI

CO-ORDINATES OF THE LUMINOSITY-PERIOD CURVE

Logarithm of Period	Absolute Visual Magnitude	Logarithm of Period	Absolute Visual Magnitude
−0.6..........	−0.34	+0.8..........	−2.43
−0.5..........	0.33	0.9..........	2.79
−0.4..........	0.3[illegible]	1.0..........	3.15
−0.3..........	0.34	1.1..........	3.51
−0.2..........	0.38	1.2..........	3.87
−0.1..........	0.50	1.3..........	4.23
0.0..........	0.64	1.4..........	4.59
+0.1..........	0.81	1.5..........	4.95
+0.2..........	0.99	1.6..........	5.31
+0.3..........	1.17	1.7..........	5.67
+0.4..........	1.37	1.8..........	6.02
+0.5..........	1.58	1.9..........	6.38
+0.6..........	1.81	2.0..........	6.74
+0.7..........	−2.10	+2.1..........	−7.1

two hundredths of a magnitude. Ten plotted points lie on the curve, 23 are below, and 24 above. The average unweighted deviation is ± 0.13 mag.,[1] an amount so nearly of the same order as the errors of the measured magnitudes that for typical[2] Cepheids of given period a rigorously constant median magnitude may be assumed. Almost all of the large deviations from the curve are of

[1] The observational errors in the periods are relatively negligible.

[2] The word "typical" is frequently used to make allowance for such stars as RV Tauri, κ Pavonis, and V Ursae Minoris, which show some general Cepheid characteristics, but because of various recognized irregularities or peculiarities may also be irregular in the relation of period to absolute brightness.

low weight, due to uncertain estimates of magnitude or period, peculiarities in light variation, or possible error in the magnitude scale. Above absolute magnitude -5.5 the curve depends only on the longest period variables in the Small Magellanic Cloud; but the resulting uncertainty is not serious, as few Cepheids have periods longer than forty days.[1]

Future work on the periods and magnitudes of variables in clusters is not likely to alter appreciably the form of the luminosity-period curve; but further investigation of the proper motions of Cepheids may contribute to the certainty of the zero-point, which is now defined both as to amount and accuracy by equation (7). The distances of even the nearest Cepheids are so great that little can be expected from direct parallax measures in the way of quantitative confirmation or improvement of the curve.[2]

IV. THE MEDIAN MAGNITUDE OF CLUSTER-TYPE VARIABLES

The flattening of the luminosity-period curve for magnitudes fainter than -0.5 indicates that for the typical cluster-type variable the median brightness is essentially invariable and is independent of the length of period. As we shall presently relate the magnitude of these variables to the maximum brightness attained in clusters, a further examination of the dispersion of median magnitudes is advisable. From preceding tables we derive the data in Table XII.

Thus the absolute *photographic* magnitude[3] for 183 variables is

$$\text{Median} = -0.23 \tag{8}$$

The deviations from this mean value may be due largely to the errors in the magnitudes of the longer-period Cepheids of each cluster. No marked change of photographic brightness with period appears; the change of visual magnitude with period

[1] Cf. Table II of the eighth paper. Long-period Cepheids are liable to irregularity.

[2] See sec. II, above.

[3] This value, which becomes of much importance in the determination of cluster parallaxes, is independent of the earlier transformations from photographic to photovisual magnitudes with the aid of Table VI.

recorded in Fig. 1 and Table XI is due to the small progression of color with period, assumed in the reductions on the basis of Fig. 2.

Much work bearing on the constancy of the median magnitude of cluster-type variables has been done at Mount Wilson; but a

TABLE XII

Cluster	Number of Variables	Mean Period	Mean Absolute Photographic Magnitude	Deviation
ω Centauri..	34	$0^{d}.40$	−0.19	$-0^{M}.04$
	37	0.59	−0.25	+0.02
	19	0.76	−0.26	+0.03
Messier 5...	8	0.27	−0.20	−0.03
	10	0.48	−0.20	−0.03
	10	0.55	−0.20	−0.03
	10	0.63	−0.22	−0.01
Messier 3...	1	0.32	−0.20	−0.03
	18	0.50	−0.31	+0.08
	18	0.54	−0.20	−0.03
	18	0.60	−0.20	−0.03

full discussion of the data would be too extensive for the present paper, and the results for only two systems are outlined below.

1. In ω Centauri three subclasses of cluster-type variables are recognized. Omitting those for which the classification is uncertain, we have in Table XIII a summary of the data bearing on the

TABLE XIII

VARIABLE STARS IN ω CENTAURI

Subclass	No. of Variables	Mean Period	Maximum Magnitude	Range of Variation	Median Magnitude	Average Deviation
a...........	33	$0^{d}.586$	12.99	$1^{M}.11$	13.55	$\pm 0^{M}.09$
b...........	15	0.752	13.10	0.87	13.55	±0.10
c...........	28	0.395	13.33	0.56	13.61	±0.09
All.........	76	....	....	....	13.57	±0.10

dispersion of median magnitudes. Although the stars of the three groups differ from each other in maximum magnitude and range, as well as in period and form of light-curve, the median values are the same. The distribution of individual deviations agrees closely with the probability curve, as shown in Table XIV.

2. The variables in Messier 3 differ very little from each other in any respect. The comparison stars and some of the variables have been studied by Miss Davis and the writer on a series of 65 photographs; the magnitude scale has been revised and referred to the Mount Wilson system. A sample of the revised data, showing the constancy of the median magnitudes, has been given in *Mt. Wilson Communication* No. 47.[1] For the 54 light-curves

TABLE XIV

Number of Residuals between	Theory	Observation
$0^{M}.00$ and $0^{M}.03$.....	18	19
0.04 " 0.06.....	14	12
0.07 " 0.09.....	13	15
0.10 " 0.12.....	10	13
0.13 " 0.15.....	7	7
0.16 " 0.18.....	5	0
0.19 " 0.24.....	6	7
$\geq$ 0.25.........	3	3

selected by Bailey as most definite the median photographic magnitude is 15.49 ± 0.01, the average deviation from the mean value is ± 0.07, and the largest deviation is less than two-tenths of a magnitude. If we include all 110 variables for which periods are known, the mean is 15.50 ± 0.006, with an average deviation ± 0.08.

A further examination of the median magnitudes for the 54 selected stars shows the following small systematic variation, which is definitely connected with the range:

Mean range of variation......	$1^{M}.04$	$1^{M}.14$	$1^{M}.26$	$1^{M}.33$	$1^{M}.48$
Mean median magnitude......	15.58	15.52	15.49	15.47	15.41
Number of variables.........	10	4	22	9	9

One explanation of this variation is that the duration of exposure was often so long that for some stars the brightness at the top of the sharp-pointed curves was never determined, the measures yielding fainter, more rounded maxima than actually exist. The range

[1] *Proceedings of the National Academy of Sciences*, 3, 480, 1917.

deduced for such stars is always too small, and the systematic error goes into the median magnitude with half its weight. The variation may be due, rather, to errors of observation at maximum light, where usually only a few estimates are available and any error will directly correlate range and median magnitude. The first explanation gives 15.4 as the true median magnitude, the second leaves it at 15.5.[1]

In either case an appropriate and simple correction to the deviations from the mean median magnitude, for the systematic errors in the maxima, leaves the uncertainty in the median magnitude but half as great as given above, and the average deviation for a single star is less than ±0.05. The distribution of the corrected residuals again accords with the law of error as closely as could be expected for a small number of values:

Number of Residuals between	Theory	Observation
$0^{M}.00$ and $0^{M}.04$	29	27
0.04 " 0.08	16	15
0.08 " 0.12	7	9
>0.12	2	3

The magnitudes in Messier 2, 5, 15, and 22 yield results similar to the foregoing. In each cluster, apparently, the total light variation of short-period Cepheids is confined to a narrow interval of brightness; and in all cases where the observations are sufficient to justify a conclusion the deviations of the median magnitudes from their mean are far within the errors of observation. Hence we are led to place much confidence in the hypothesis that the parallax of a cluster-type variable (or of any cluster containing such stars) may be derived immediately from the measurement of the median magnitude.

[1] The error is probably common to all groups of variables. It does not vitiate the work on cluster parallaxes, for the median value as observed is used to obtain both apparent and absolute magnitudes. A systematic error may be introduced into the determination of the distances of some isolated cluster-type variables, but at most it will be only a few per cent and far within the uncertainty of the various magnitude scales.

V. PARALLAXES OF CLUSTERS FROM THEIR BRIGHTEST STARS

The most important use of accurate median magnitudes is to furnish a starting-point for the study of the absolute luminosity of the brightest stars in a cluster. In Messier 3, for instance, where the magnitudes of all the brighter stars[1] have been accurately measured, there are nearly 300 more luminous than the median magnitude of the variables. A few that are more than two magnitudes brighter appear in the cluster, it may be, by projection only,

TABLE XV

PHOTOGRAPHIC MAGNITUDES OF 30 BRIGHTEST STARS IN MESSIER 3

Star	Color-Index	Pg. Mag.	Deviation	Star	Color-Index	Pg. Mag.	Deviation
206....	+1.01	11.27	Bright	1449....	+1.40	14.18	−0.05
420....	+0.19	13.59	Bright	463....	+1.13	14.25	+0.02
589....	+0.91	13.71	Bright	238....	+1.79	14.27	+0.04
205....	+1.09	13.81	Bright	334....	+1.13	14.27	+0.04
837....	+1.36	13.83	Bright	265....	+1.14	14.29	+0.06
				925....	+1.34	14.29	+0.06
1127....	+1.30	13.92	−0.31	1397....	+1.69	14.29	+0.06
1219....	+1.40	14.04	−0.19	398....	+1.13	14.32	+0.09
706....	+1.16	14.06	−0.17	237....	+0.42	14.33	+0.10
1000....	+1.25	14.08	−0.15	309....	+1.19	14.37	+0.14
1128....	−0.60	14.08	−0.15	640....	+1.18	14.40	+0.17
417....	+1.15	14.09	−0.14	1203....	+1.32	14.40	+0.17
740....	+0.70	14.70	−0.13	1208....	+1.23	14.40	+0.17
490....	+1.80	14.13	−0.10	1214....	+1.22	14.40	+0.17
297....	+1.32	14.14	−0.09	177....	+1.00	14.45	+0.22
1392....	+1.22	14.14	−0.09				
				Means..	+1.16	14.23	±0.12

or they may be double or multiple stars. If we limit our study to the region within 9′ from the center and exclude a few, say five, of the very brightest objects, we can feel sure that practically all the remaining bright stars are really typical members of the cluster.

In Table XV the sequence of the 30 brightest objects is shown for Messier 3, the data being taken from an unpublished investigation of the magnitudes and colors of nearly a thousand stars.[2] The

[1] Stars within a concentric circle of nearly 3′ diameter are excluded because of possible systematic errors arising from crowding of images and Eberhard effect.

[2] The numbers are those of von Zeipel's catalogue, *Annales de l'Observatoire de Paris*, **25**, 1908.

mean luminosity of the 25 chosen stars is determined with nearly the same accuracy as the median magnitude of the variables. The difference, Med. − Mean Br., is $+1.27$; hence from equation (8) we find that the brightest stars have an absolute photographic magnitude of -1.5 in the mean, and a maximum just fainter than -2. These bright stars are reddish, however, and the maximum visual absolute brightness exceeds -3 magnitudes in a few cases.

Other clusters which contain short-period Cepheids agree in showing that the maximum photographic luminosity is regularly between 1.5 and 2.0 magnitudes brighter than the median value for the variables. We see, therefore, in this apparent constancy of maximum magnitude, the possibility of an expeditious method of furthering our knowledge of cluster distances. The phenomenon is qualitatively illustrated by an inspection of the photographs reproduced in *Harvard Annals*, **38**; and a quantitative confirmation is readily possible through magnitude measures, even when the variables are few in number and their light-curves are unknown. Thus, for Messier 22 a study of three polar-comparison plates gives for the mean magnitude of the 25 brightest stars[1] the value 13.08, with extremes of 12.51 and 13.55, and an average deviation of ± 0.19, while the median magnitude of the variable stars,[2] from the measures given in Table XVI, appears to be 14.45. Hence, Med. − Mean Br. = $+1.37$. For a few clusters, in which the variables have been extensively studied, more accurate values of this difference can be obtained.

All material now available has been considered in discussing the relation of median to brightest magnitude, summarized in Table

[1] Five stars brighter than the "25 brightest" are always excluded in using this method.

[2] Sixteen variable stars are listed by Bailey (*Harvard Annals*, **38**, 242, 1902). No. 11 is a very close double and was not measured; Nos. 3 and 14 could not be certainly identified. The variability of Nos. 2, 5, 9, and 16 is not definitely confirmed by the Mount Wilson plates. No. 8 is one of the brightest stars in the cluster and appears to be a long-period Cepheid; similarly, Nos. 5 and 9 are probably bright Cepheids of long period. For the remaining 8 stars a short-period variation of a magnitude or more is fairly well established by these plates. The total interval of brightness is about two magnitudes; the uncertainty of the median magnitude given above is possibly one or two tenths..

XVII.[1] Some of the tabulated material has been adapted from Bailey's published work, but most of it has been derived from Mount Wilson plates, of which the total number entering the discussion for each cluster is noted in the second column. The third column gives the number of cluster-type variables contributing to the mean median magnitude. It should be observed that the final result appears to be independent of the frequency of variable stars.

TABLE XVI

VARIABLES IN MESSIER 22—OBSERVATIONS OF 1917

Number	Plate 3887 Aug. 14.71	Plate 3888 Aug. 14.72	Plate 3892a Aug. 14.75	Plate 3892b Aug. 14.75	Plate 3940 Sept. 9.70	Plate 3963 Sept. 10.68
1..........	....	....	....	15.51	14.06	15.37
2..........	14.96	....	....	15.12	15.15	14.89
3..........	....	....	....	....	....	16.36?
4..........	14.80	15.06	....	14.95	15.13	14.38
5..........	12.72	12.78	12.72	12.75	12.46	12.64
6..........	15.04	14.79	....	14.37	14.98	14.01
7..........	15.06	15.10	....	14.78	14.32	15.41
8..........	12.16	12.17	12.29	12.10	12.76	12.79
9..........	13.24	13.31	13.21	13.17	13.32	13.15
10..........	15.14	15.02	....	15.39	15.03	14.38
12..........	15.01	14.99	....	14.42	14.95	14.93
13..........	14.20	13.78	13.72	13.58	14.98	13.68
15..........	14.95	14.99	....	15.36	14.52	14.93
16..........	14.41	14.54	....	14.52	14.47	14.34

The radius of the concentric circular area in which the bright stars were measured is given in the fourth column of Table XVII. The choice of this quantity is somewhat arbitrary, and small changes in it may affect the mean perceptibly if thereby exceptionally bright stars happen to be included or omitted.[2] The adopted radius

[1] In at least three clusters (N.G.C. 6266, 6626, and 6723), in which magnitudes have not been quantitatively studied, a considerable number of short-period variable stars are from one to two magnitudes fainter than the brightest stars. Qualitatively, therefore, the relation between the median and brightest magnitudes is verified in 10 clusters and the Magellanic clouds and is nowhere controverted. Its quantitative expression, however, is probably less definite than the differences of Table XVII suggest, the close agreement of several values being partially fortuitous. A later study of Messier 15, for instance, indicates an uncertainty of 0.2 mag. in the provisional value given here; but the adopted probable error of the mean difference amply covers this discrepancy.

[2] The mean magnitude for Messier 3 is 14.30 for a radius of 7′ and 14.17 for a radius of 11′.

depends in general upon both the nature of the photographs and the cluster's angular diameter, but mainly upon the galactic latitude. In rich fields the area is necessarily small in order to exclude bright non-cluster stars. We might have adopted the same radius for all clusters and attempted to allow for foreign objects by varying with galactic latitude the number of excluded stars, but the actual angular dimensions differ so greatly that a more flexible procedure seemed advisable. Every effort has been made in this and subsequent work of the same kind to obtain homogeneous results, in each case so choosing the area for measures of bright stars that, with the five brightest rejected, the resulting mean gives a trustworthy value of the maximum luminosity.[1]

TABLE XVII

MEDIAN MAGNITUDES AND THE BRIGHTEST STARS

Cluster	No. of Plates	No. of Variables	Radius	Photographic Magnitude			Weight
				Mean Br.	Median	Diff.	
Messier 3..	65	110	9′	14.23	15.50	1.27	8
5..	3	61	4	13.97	15.26	1.29	4
15..	7	48	6	14.31	15.63	1.32	2
2..	7	7	4	14.61	15.71	1.10	1
22..	6	8	5.5	13.08	14.45	1.37	1
13..	15	4?	6	13.75	15.3:	1.5:	0
ω Centauri.	3	90	....	12.3:	13.9:	1.6:	0
				Mean difference....		1.28	

The mean difference, $+1.28$, combined with (8), gives $M = -1.51$ as the mean absolute photographic magnitude for the bright stars. The probable error of the difference is not likely to be in excess of two-tenths of a magnitude. This estimate makes generous allowance for the real dissimilarities of the clusters (which seem to be small so far as magnitude limits go) and for the uncertainties in excluding peculiar and non-cluster stars. Taking also into consideration the probable error of (7) and the observational errors in the apparent magnitudes, m, we conclude that the

[1] Central condensations and multiple stars were avoided. Counts of stars on the Franklin-Adams charts were frequently of service in estimating the probable frequency of outside stars in a cluster field.

probable error for the difference $M-m$, in the relation $5 \log \pi = M-m-5$, certainly does not exceed 0.4 mag., and, therefore, that the absolute parallax of a globular cluster may be obtained from the apparent magnitudes of its brightest stars with a probable error of less than 20 per cent. For relative parallaxes the probable error does not include the error of (7) or the possibility of systematic error in the magnitude scale; its value is 10 per cent or less for parallaxes derived from homogeneous data.

VI. REMARK ON THE PARALLAXES OF CLUSTERS DERIVED FROM THEIR APPARENT DIAMETERS

The foregoing discussion shows that the mean apparent magnitude of the brightest stars in a globular cluster is a pretty dependable criterion of its distance, thus indicating that all systems are much alike in the maximum luminosity attained by any individual member. In consequence it is a natural assumption that clusters may also be closely comparable in actual size. In fact, the first paper of this series[1] contains a provisional curve correlating decreasing maximum brightness with decreasing angular diameter, and it follows that the apparent size of a globular cluster is also a direct measure of the parallax. As we may obtain the parallaxes of nearly 30 globular clusters by the methods outlined on preceding pages, a curve showing the relation of distance to apparent size can be readily constructed, and using this curve the parallax of any other cluster can be obtained from its diameter. A necessity of such work is homogeneity in the observations, and this is afforded in a highly satisfactory manner by the Franklin-Adams photographic charts, which cover the whole sky and include every known globular cluster. Further discussion of this phase of the work is reserved for the following contribution.

VII. SUMMARY

1. The determination of the distances and distribution in space of globular clusters involves a general treatment of extensive data bearing on the magnitudes, periods, light-curves, proper motions, and radial velocities of Cepheid variables in the Galaxy

[1] *Mt. Wilson Contr.*, No. 115, p. 12, 1915.

and other systems and on the angular diameters of clusters and the number, magnitudes, and colors of their individual stars. Its successful accomplishment will help somewhat to a better understanding not only of the most remote objects known in the stellar universe, but also of the dimensions and dynamics of cluster systems and of the maximum luminosity attainable in stellar evolution. Suggestions relative to the extent and arrangement of the galactic system and to the sun's position therein will be a natural outcome of the work.

2. From parallactic motions the mean absolute magnitude of eleven isolated Cepheid variable stars has been derived with a relatively small computed probable error (sec. II). The luminosities of the individual stars are shown to be uniquely defined by their periods.

3. An extension of these results gives a relation (Fig. 1) connecting the periods of both the ordinary Cepheids and the cluster-type variables with their absolute magnitudes, which permits the derivation of the distances of all such variable stars as soon as their periods and apparent magnitudes are measured; and when we adopt the plausible hypothesis that Cepheids of a given period are comparable wherever found, the relation also yields the parallax of any cluster containing Cepheid variables. Data for more than 200 individual variables from seven different stellar systems contribute to the determination of the luminosity-period relation. Fainter than a definitely fixed luminosity Cepheid variation probably never occurs.

4. Further investigation makes the derivation of cluster parallaxes practically independent of variable stars by substituting the apparent magnitudes of the brightest stars as the criteria of distance (sec. V). Stars brighter than the absolute photographic magnitude -2 are exceedingly rare in clusters.

5. Angular diameters are next employed in extending the work, until finally for all globular clusters in both hemispheres values of the parallax become possible. The distances are derived and considered statistically in the next paper of this series.

Mount Wilson Solar Observatory
November 1917

THE ABSORPTION OF NEAR INFRA-RED RADIATION BY WATER-VAPOR

By W. W. SLEATOR

I. INTRODUCTION

The study of the absorption of radiation in the atmosphere was begun by Langley,[1] whose bolographs, or maps of the sun's spectrum, show wide bands attributed by him to water-vapor. Paschen,[2] in work on the emission of radiation by gases, has investigated as well the absorption occurring in steam, and records in particular the effect of changes in temperature upon the wave-lengths of the radiation absorbed. His work has been extended by Eva von Bahr,[3] who has shown that the doublet between wave-lengths 5 and 7 μ is in reality very complex and contains about forty separate absorption bands arranged with some appearance of symmetry on either side of the wave-length 6.26 μ.

The combination principle suggested by Bjerrum,[4] in connection with his application to the case of molecular rotation of the quantum theory, is a fruitful hypothesis in the explanation of near infra-red absorption. Bjerrum's ideas have in fact been applied by Eucken[5] to the system of water bands presented in the work of Eva von Bahr, and that system seems to be accounted for in terms of two series of rotation-frequencies corresponding to two principal moments of inertia in the water molecule. More recent work by Rubens and Hettner[6] shows some evidence for the presence in the farther infra-red of a third series of rotation-frequencies. The

[1] *Researches on Solar Heat* (Professional Papers of the Signal Service, No. 15, United States War Department).

[2] *Annalen der Physik*, **51**, 4, 1894; **52**, 210, 1894; **53**, 234, 1894.

[3] *Phil. Mag.*, **28**, 71, 1914; *Verhandlungen der deutschen Physikalischen Gesellschaft*, **15**, 731, 1913.

[4] *Nernst Festschrift*, p. 90.

[5] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **15**, 1159, 1913; *Phil. Mag.*, **28**, 71, 1914.

[6] *Chemical Abstracts*, **11**, 1357, 1917; *Science Abstracts*, No. 234, p. 223, June 1917.

question of molecular structure is now intimately connected with the investigation of radiation, as is shown by further work of Bjerrum,[1] and important tests of the quantum hypothesis await experimental advances in the same field.[2]

The present paper gives an account of work undertaken to extend our knowledge of atmospheric absorption. It concerns particularly the region of the spectrum between the wave-lengths 1.3 and 3 μ and presents a detailed study of the absorption regions marked Ψ, Ω, and X on Langley's charts.

II. APPARATUS

The work of Eva von Bahr was done with a prism spectrometer and radiomicrometer. In the work recounted here a higher dispersion was secured by the use of gratings, and the uncertainty of wave-lengths determined with the aid of a dispersion-curve was at the same time avoided. The troublesome overlapping of grating spectra has been eliminated by the use of a supplementary prism. The general arrangement is shown in Fig. 1. The apparatus takes the form of two spectrometers, each with fixed arm and single mirror.[3] By a proper setting of the prism a definite and limited portion of the spectrum is presented to the grating spectrometer. The prism serves in fact as an effective screen or filter.

A linear thermopile, incased at T in Fig. 1, and a galvanometer of the Paschen type constituted the detecting system. The ballistic deflections of the galvanometer observed on opening or closing the shutter at O serve to map out the spectrum of the Nernst glower used as the source of energy. The construction of the galvanometer was the first work done on the present problem.

The entire path of the light, shown in Fig. 1 by the dotted lines, is inclosed in a black box of double cardboard. It is of course not possible to make such a box tight, nor to get the air in it absolutely dry. In the section around the glower, however, the air could be saturated. Also energy-curves were plotted from data secured on winter nights, when cold air from out of doors was pumped through

[1] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **16**, 737, 1914.

[2] *Ibid.*, **16**, 614, 1914; **15**, 1159, 1913.

[3] *Annalen der Physik*, **33**, 739, 1910.

sulphuric acid and phosphorus pentoxide into the cases. Sufficient difference appeared between the depths of the bands to enable us to attribute the absorption to water-vapor.

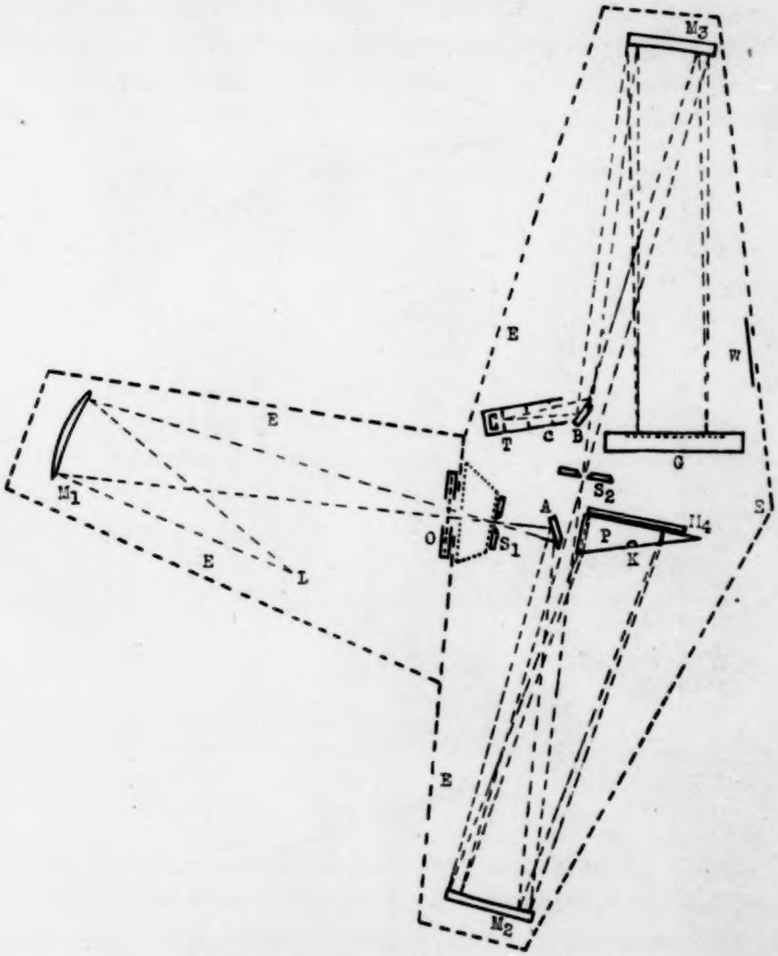

FIG. 1.—The spectrometer, scale 1 to 5

L, Nernst glower; S_1, S_2, slits; M_1, 10 cm mirror, f = 20 cm; *P*, salt prism; M_2, M_3, 10 cm mirrors, f = 50 cm; M_4, *A*, *B*, plane mirrors; *G*, grating; *C*, case for *T*, the thermopile; *W*, window in box *E*; *O*, shutter. The path of the light is $LM_1S_1AM_2PM_4PM_2S_2M_3GM_3BT$. A spectrum appears at S_2. *P* and M_4 rotate together about K, so that any region of the spectrum may be isolated for the grating, and the overlapping of spectra is avoided.

In the region of the spectrum near the wave-length $2.6\ \mu$ some work has been done with steam. The absorption took place in a chamber 15 cm long, which was kept hot by resistance coils, and could be swung into or out of the beam of light. The glass ends of the chamber were compensated for by two pieces cut from the same

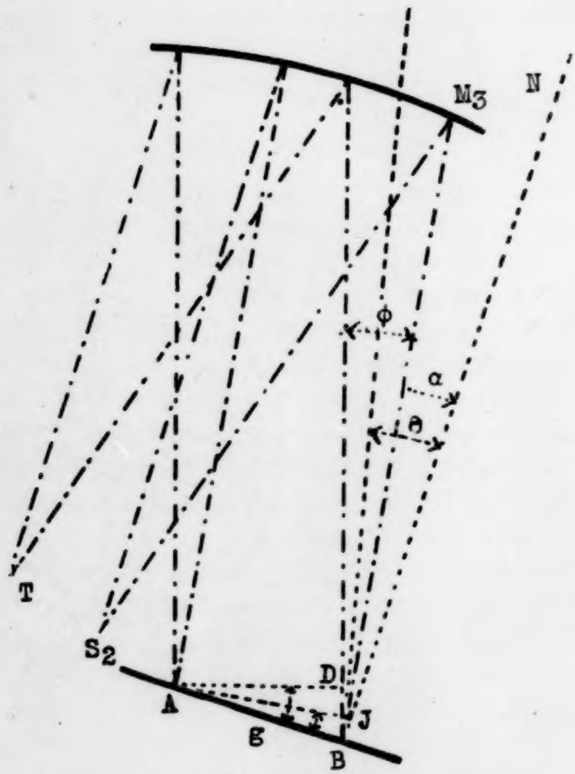

FIG. 2.—The spectrometer constant κ in the equation $\lambda=\kappa \sin\theta$

AB is the grating space, *g*. Light from S_2 is sent to *T*. *T*, S_2, and M_3, the mirror, are fixed, and rotation of the grating leaves ϕ unchanged. The wave-length at *T* is determined by the relation

$$\begin{aligned} n\lambda &= DB+BJ \\ &= g \sin(\alpha+\phi) + g \sin\alpha \\ &= 2g \sin(\alpha+\tfrac{1}{2}\phi) \cos\tfrac{1}{2}\phi \\ &= 2g \cos\tfrac{1}{2}\phi \sin\theta \\ &= \kappa \sin\theta \end{aligned}$$

$\theta=\frac{1}{2}(R_1-R_2)$, R_1 and R_2 being circle readings on the two spectra of one order.

sheet of glass, through which the radiation passed when the steam chamber was drawn aside.

Three gratings have been employed in the course of the work. The smallest has a ruled surface 5 cm wide and about 2400 lines to the inch, and is referred to as the brass grating. The other two have effective surfaces 12.2 cm wide, are ruled on speculum metal, and are referred to as the 7500-line grating and the 15,000-line

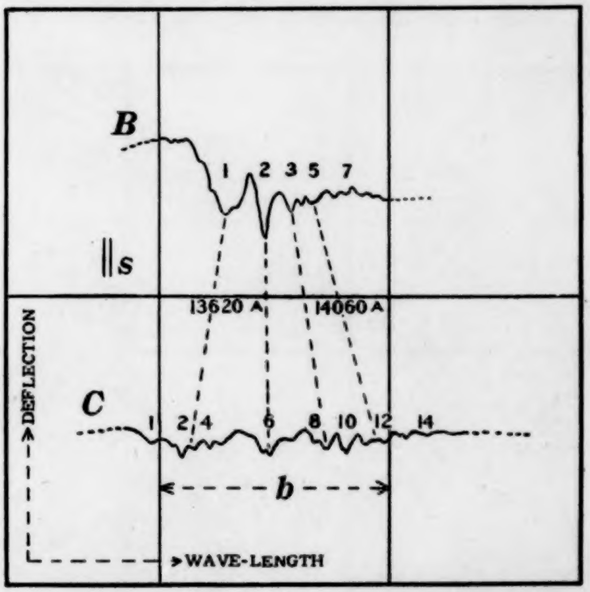

FIG. 3.—Energy-curves, region of 1.3 μ. The slit corresponds to 33 A in *B* and 15 A in *C*.

grating, the figures giving the number of lines per inch. In Figs. 3, 4, 5, and 6 the letters *A*, *B*, and *C* refer respectively to these gratings. They are mounted on the table of a Schmidt and Haensch spectrometer whose circle may be read to ten seconds.

Fig. 2 serves to illustrate the meaning and derivation of the equation

$$\lambda = \kappa \sin \theta,$$

used in computing wave-lengths from data secured with the fixed-arm spectrometer. To obtain the wave-length interval included at

one time in the slit of the thermopile, one considers first the resolving power of the grating. If the source S_2 were very narrow, the width of the central bright band in the diffraction pattern produced by monochromatic light would be given by

$$w = \frac{2\lambda}{b \cos \theta} \cdot 500,$$

where b is the width of the grating in mm, θ the angle by which the grating is turned from the normal, and 500 the focal length of the

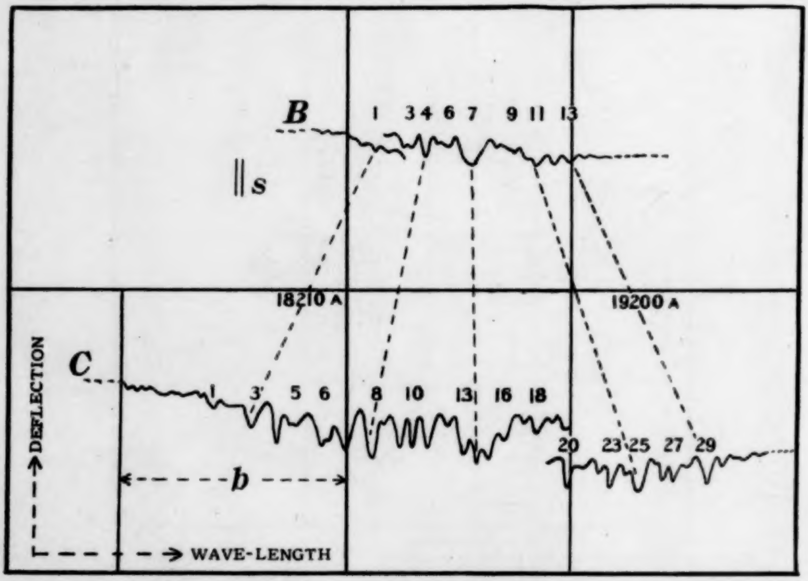

FIG. 4.—Energy-curves, region of 1.8 μ. The slit corresponds to 33 A in B and 14 A in C.

mirror in mm. For the 15,000-line grating w is 0.03 mm at 2.6 μ, and for the brass grating 0.14 mm at 6 μ. The slit S_2 is $\frac{1}{2}$ mm wide, and its image in the plane of the slit of the thermopile is in the first case about 0.52 mm and in the second about 0.57 mm wide. With the slits now in use no great improvement in the purity of the spectrum can be attained by enlarging the grating.

The range of spectrum included in the slit is taken as the interval in wave-lengths between two beams whose centers are $\frac{1}{2}$ mm apart

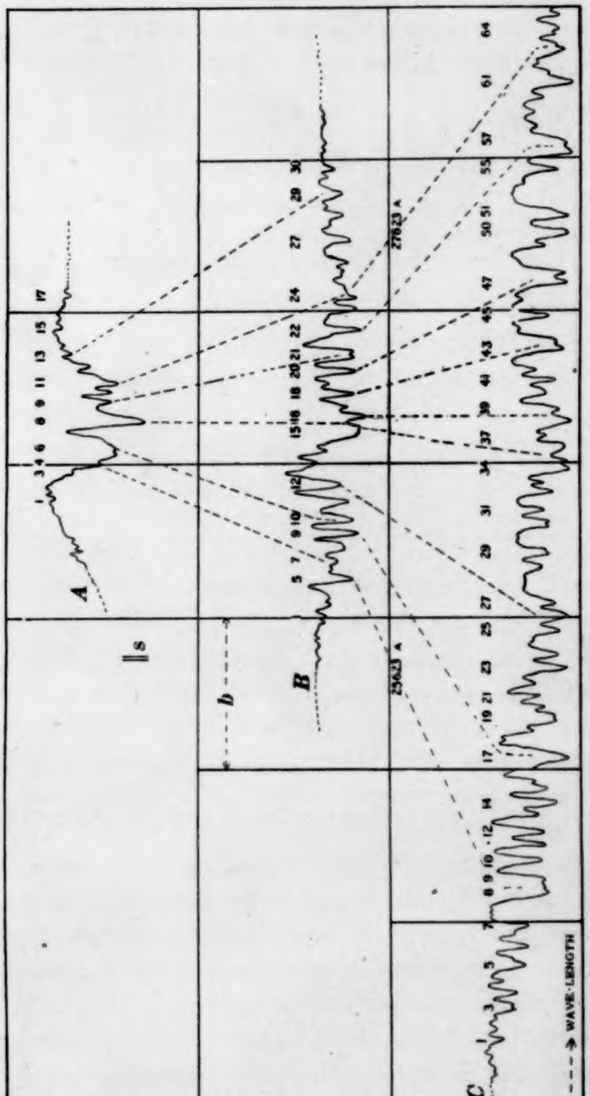

FIG. 5.—Energy-curves, region of 2.6 μ. The slit corresponds to 105 A in *A*, to 31 A in *B*, and to 11 A in *C*. *C* does not give the entire curve of this region as explored with the 15,000-line grating.

in the plane of the thermopile. That slit does not take in the total energy even of a single wave-length, but does receive about three-fourths of the energy in the interval named. This interval corresponds to a variation in ϕ (in Fig. 2) of $\frac{1/2}{500}$ or 0.001 radian. Taking the equation for wave-length as

$$\lambda = 2g \cos \phi/2 \sin (\alpha + \phi/2),$$

and considering that $\phi/2$ is found by measurement to be $1^\circ\ 16'$, we have by differentiation the approximation

$$d\lambda = K \cos \theta d\theta$$

$$= K \cos \theta \frac{d\phi}{2}.$$

This expression gives the values set down with the figures, which are a little too large. In order to throw a single beam from one

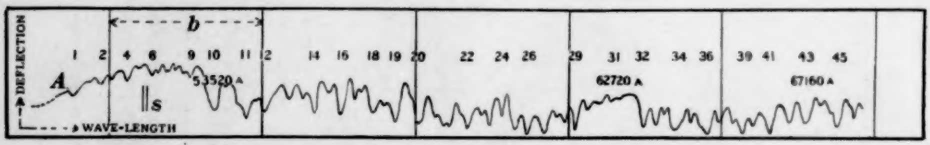

FIG. 6.—Energy-curve, part of the region of 6 μ. The slit corresponds to 100 A

side of the slit of the thermopile to the other, ϕ must be changed by $\frac{1/2}{500}$ or 0.001 radian, as above, and this demands a change in α of 0.0005 radian and a corresponding motion of the grating. 0.0005 radian is $1'.7$, and the spectrum interval may be checked by noting the value of $1'.7$ in units of wave-length along an energy-curve. At 2.6μ the 15,000-line grating gives this interval as 10.4 A.

The green line of the spectrum of mercury ($\lambda = 5460.74$ A, Kaye and Laby's tables) was used for visual calibration, the bright line and the slit of the thermopile being viewed objectively. In the case of the brass grating no visible lines could be observed, but the actual grating-space was known, and

$$K = 2g \cos \tfrac{1}{2}\phi$$

was computed from data obtained from measurement.

III. THE CURVES

Figs. 3, 4, 5, and 6 present a summary of the experimental work. Except in Fig. 6 each energy-curve is one of three secured, for two of them the air being saturated and for one dried. The bands are numbered arbitrarily for reference to the tables. The letters *A*, *B*, and *C* refer respectively to the brass, the 7500-line, and the 15,000-line grating. In Fig. 7, as well as in the others, the distance *b* denotes 1° on the circle and *s* shows the relative size of the thermopile slit.

Fig. 7 represents part of the region at $2.6\ \mu$ as mapped with the 7500-line grating. The upper curve (lettered *D*) does not show the galvanometer deflections but gives the percentage of incident energy transmitted by steam. The effect of drying the air appears also in this figure, in curve *E*, and it will be noted that the curve for steam accentuates the peculiarities of *F*, the "air-saturated" curve, peculiarities which the curve secured with dried air decidedly obscures. It seems that the temperature of the absorbing vapor has no effect on the position of an individual band. According to Paschen's work, already cited, we conclude that the relative intensities of the bands must change, so that those that are deepest at one temperature are not deepest at another. But the present work cannot confirm nor deny this effect. The effect on the absorption of drying the air, shown in Fig. 7, was found even more definitely with the finer grating, at 1.3 and 1.8, as well as at $2.6\ \mu$. All the deeper bands are less marked when the air is dry. This does not appear so clearly with the shallow bands, but it may be that in the "air-saturated" curves the presence of weak bands is masked by their more prominent neighbors. It is upon such evidence as appears in Fig. 7 that we ascribe the absorption to water-vapor.

With the 7500-line grating the intervals of the spectrum between the absorption bands at 1.3 and $1.8\ \mu$, and between those at 1.8 and $2.6\ \mu$, were carefully explored and were found to present no evidence of atmospheric absorption.

In Figs. 3 and 5 the upper curves are decidedly similar. This similarity suggests that one of these absorption regions may be a harmonic of the other in the sense suggested by Kemble.[1] However,

[1] *Physical Review*, **8**, 689, 1916.

a frequency half that corresponding to band No. 2 in Fig. 3 corresponds nearly to band No. 11 in Fig. 5 rather than to band

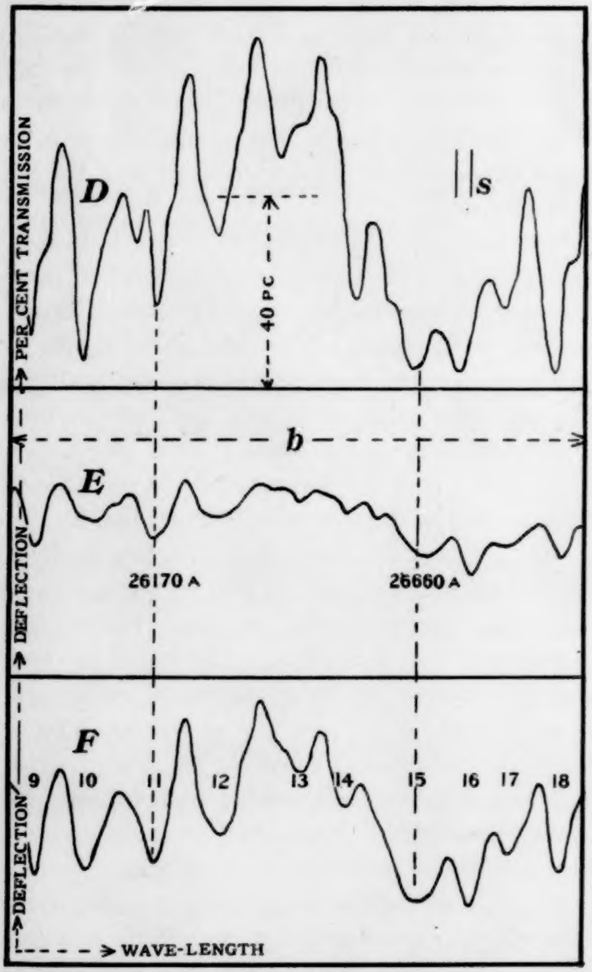

FIG. 7.—Part of the region of 2.6 μ, 7500-line grating. *D* gives the percentage of energy transmitted by steam at 110 to 120° C. and 1 atmosphere pressure. *E* is an energy-curve with the air partly dried, *F* an energy-curve with the air saturated at about 35° C.

No. 8, and the same lack of agreement appears in the lower curves. Evidently the simple idea of harmonics does not apply.

A comparison of the energy-curves in Fig. 5, secured respectively with the 7500- and 15,000-line gratings, shows how the increased dispersion adds to our knowledge of the absorption. Bands No. 8 and No. 9 in the lower curve, for example, appear in the middle one as No. 5, a connection indicated in typical cases by the dotted lines, and appearing more definitely in the tables. These curves must be considered in any numerical computations made with absorption frequencies, for in the curves appear the individualities of the bands, which cannot well be expressed in numbers. In all cases the grating was turned 1′ for each new setting. At 6 and at 2.6 μ six deflections were taken at each minute, and at 1.3 and 1.8 μ four. The computations are based on curves which, for the region at 2.6 μ, are about six feet long.

IV. THE TABLES

On pages 137 to 142 are set forth the wave-lengths and frequencies of the bands, which are denoted by the numbers given with the curves. Corresponding wave-lengths appear opposite each other. The gratings have been calibrated independently, the values given have been calculated from independent curves, and the agreement between opposite numbers is in general satisfactory. In connection with the region of 6 μ is set forth some evidence of the symmetry demanded by Eucken's explanation, but unfortunately the work was not carried far enough to give this evidence much value. Some idea of the accuracy of the determinations of λ and N might be obtained by estimating the effects of various sources of error. But first one may consider the different values secured for the same quantity from the three corresponding curves. The "typical computations of λ and N" show these numbers for certain bands in the region at 2.6 μ—bands taken quite at random. Considering the uncertainty in the value of K we should perhaps affect the wave-length by the probable error of ± 2 A. Errors in the circle of the spectrometer are probably negligible, especially since the calibration involved the same sections of the circle as were employed in the measurements. At 6 μ, where we have only one curve, obtained under less favorable circumstances, we should perhaps write the wave-lengths with an error ± 20 A. A question mark in the tables

TYPICAL DATA

The settings of the grating circle, and corresponding deflections of the galvanometer.

2 A.M., March 22, 1917. Grating temperature, 27°. Period of galvanometer, 6 secs.; E.M.F. applied to glower, 238 volts.

Circle	Deflections in mm	Circle	Deflections in mm
121° 10′......	28–9– 9–8–9–9 = 28.7	120° 58′.....	18– 9– 8–9–9–9 = 18.7
9′......	26–7– 7–6–7–6 = 26.5	57′.....	19–20– 0–0–0–0 = 19.8
8′......	23–2– 1–2–2–2 = 22.0	56′.....	16– 7– 8–8–8–7 = 17.3
7′......	12–5– 6–6–4–5 = 14.7	55′.....	9–11–10–1–0–1 = 10.3
6′......	11–2– 3–2–3–1 = 12.0	54′.....	10– 0– 1–0–1–1 = 10.5
5′......	10–2– 2–1–2–2 = 11.5	53′.....	18–22– 0–1–0–1 = 20.3
4′......	15–8– 7–7–8–7 = 17.0	52′.....	24– 6– 5–5–6–5 = 25.2
3′......	17–6– 6–7–6–7 = 16.5	51′.....	27– 7– 6–7–7–7 = 26.8
2′......	9–8–10–9–9–8 = 8.8	50′.....	27– 8– 6–7–6–8 = 27.0
1′......	5–5– 4–5–5–6 = 5.0	49′.....	25– 7– 6–8–6–7 = 26.5
121° 0′......	6–6– 5–6–6–6 = 5.8	48′.....	23– 6– 4–6–4–6 = 24.8
120° 59′......	14–3– 4–3–4–3 = 13.5	47′.....	22– 3– 1–4–2–3 = 22.5

TYPICAL COMPUTATIONS OF WAVE-LENGTH AND FREQUENCY

REGION OF 2.6 μ

Band No.	Curve No.	λ	Mean λ	N	Mean N
20........	1	26063.4	26066.2	383.69	383.64
	2	26068.8		383.60	
	3	26066.5		383.63	
23........	1	26162.7	26161.6	382.22	382.24
	2	26161.7		382.25	
	3	26160.4		382.26	
27........	1	26321.3	26325.5	379.92	379.86
	2	26330.0		379.79	
	3	26325.3		379.86	
31........	1	26543.7	26546.4	376.74	376.70
	1′	26546.9		376.70	
	2	26549.3		376.66	
	3	26545.7		376.71	
32........	1	26587.4	26589.8	376.12	376.08
	1′	26589.5		376.09	
	2	26590.9		376.07	
	3	26591.4		376.06	
33........	1	26608.7	26609.9	375.82	375.80
	1′	26610.9		375.79	
	2	26610.2		375.80	
	3	26609.7		375.80	

denotes not a doubtful band but a doubtful correspondence. The second column in the tables for the region at 2.6 μ gives the relative depths of the bands as an indication of their importance in other infra-red work. A band numbered 1 seems to represent an absorption of less than 10 per cent, while 5 shows that perhaps 80 per cent of the incident energy is absorbed.

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM

REGION OF 1.38 μ

15,000-Line Grating			7500-Line Grating		
Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
1	13545	738.3			
2	13614	734.5	1	13620	734
3	13643	733.0			
4	13678	731.1			
5	13710	729.4			
6	13820	723.6	2	13820	723.5
7	13876	720.7			
8	13922	718.3			
9	13954	716.7	3	13950	717
10	14000	714.3	4	14010	714
11	14046	712.0	5	14060	711
12	14092	709.7			
13	14140	707.2	6	14170	706
14	14186	704.9			
			7	14240	702

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

REGION OF 1.87 μ

15,000-LINE GRATING			7500-LINE GRATING		
Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
1..........	18110	552.2			
2..........	18152	550.9			
3..........	18202	549.4	1	18210	549.1
4..........	18263	547.6	2	18287	546.8
5..........	18306	546.3			
6..........	18363	544.6	3	18398	543.6
7..........	18414	543.1			
8..........	18471	541.4	4	18467	541.5
9..........	18534	539.6	5	18530	539.7
10..........	18564	538.7			
11..........	18595	537.8	6	18590	537.9
12..........	18641	536.4			
13..........	18676	535.5			
14..........	18705	534.6	7	18704	534.6
15..........	18729	533.9			
16..........	18765	532.9			
17..........	18806	531.8			
18..........	18836	530.9	8	18836	530.9
19..........	18862	530.2			
20..........	18897	529.2	9	18912	528.8
21..........	18926	528.4			
22..........	18968	527.2			
23..........	18995	526.5	10	18987	526.7
24..........	19022	525.7	11	19038	525.3
25..........	19054	524.8			
26..........	19106	523.4			
27..........	19133	522.7	12	19126	522.9
28..........	19157	522.0			
29..........	19199	520.9	13	19195	521.0
30..........	19234	519.9	?14	19264	519.1

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

REGION OF 2.6 μ

15,000-Line Grating				7500-Line Grating			Hilger Brass Grating		
Band No.	Relative Intensity	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
1.....	1	25235	396.28				?1	24850	402.2
2.....	1	25262	395.85				?2	25040	399.4
3.....	2	25311	395.09						
4.....	2	25352	394.45	1	25347	394.52			
5.....	2	25425	393.31	2	25432	393.21			
6.....	2	25469	392.63	?3	25481	392.45	?3	25490	392.3
7.....	2	25520	391.85	4	25517	391.90			
8.....	3	25608	390.50						
				5	25623	390.27			
9.....	3	25636	390.08						
10.....	3	25688	389.29	6	25696	389.17			
11.....	3	25727	388.70						
				7	25747	388.39			
12.....	3	25760	388.20						
13.....	2	25803	387.55				4	25760	388.2
14.....	4	25830	387.15						
				8	25837	387.04			
15.....	3	25864	386.64						
16.....	2	25880	386.40						
17.....	5	25940	385.51	9	25941	385.49	5	25920	385.8
18.....	2	26008	384.50						
19.....	3	26046	383.94	10	26044	383.97	6	26080	383.4
20.....	2	26066	383.64						
21.....	2	26090	383.28						
22.....	2	26123	382.80						
23.....	3	26161	382.25	11	26168	382.15			
24.....	4	26196	381.74						
25.....	4	26256	380.87						
26.....	5	26295	380.30	12	26300	380.23	7	26280	380.5
27.....	4	26325	379.87						
27½....	1	26385	379.00						
28.....	2	26411	378.63						
29.....	3	26448	378.10	13	26445	378.14			
30.....	3	26515	377.15						
				14	26528	376.96			
31.....	2	26547	376.69						
32.....	3	26590	376.08						
33.....	2	26610	375.80						
34.....	5	26645	375.30						
35.....	5	26661	375.08	15	26663	375.05			
36.....	5	26696	374.59						
37.....	3	26720	374.25				8	26730	374.1
38.....	5	26760	373.69						
				16	26765	373.62			
39.....	4	26783	373.37						
40.....	3	26827	372.76						
				17	26837	372.62			
41.....	3	26853	372.40						
42.....	2	26880	372.02						

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

REGION OF 2.6 μ—*Continued*

15,000-Line Grating				7500-Line Grating			Hilger Brass Grating		
Band No.	Relative Intensity	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
43.....	4	26930	371.33	18	26936	371.25			
44.....	1	26974	370.73						
45.....	3	27006	370.29	19	27002	370.34			
46.....	1	27044	369.77						
47.....	4	27084	369.22						
				20	27093	369.10			
48.....	4	27103	368.96						
49.....	3	27168	368.08						
50.....	5	27198	367.67	21	27193	367.74	9	27180	367.9
51.....	5	27236	367.15						
52.....	1	27278	366.60						
53.....	1	27294	366.38						
54.....	1	27314	366.11						
55.....	3	27340	365.76						
56.....	5	27386	365.15						
				22	27395	365.03			
57.....	5	27407	364.87						
58.....	2	27445	364.37				10	27430	364.6
59.....	2	27479	363.91						
60.....	2	27511	363.49						
61.....	5	27545	363.04	23	27544	363.06			
62.....	4	27614	362.14						
63.....	3	27625	361.99	24	27623	362.02	11	27620	362.0
64.....	3	37655	361.60						
65.....	3	27668	361.43						
66.....	3	27700	361.01						
67.....	3	27712	360.85						
68.....	2	27765	360.17						
69.....	2	27777	360.01	25	27784	359.92			
70.....	2	27803	359.67						
71.....	2	27819	359.47						
72.....	3	27867	358.85	26	27866	358.86			
73.....	1	27926	358.09						
74.....	1	27947	357.82						
75.....	1	27972	357.50						
76.....	4	28029	356.77	27	28031	356.75	12	28030	356.8
77.....	2	28077	356.16						
78.....	1	28103	355.83						
79.....	2	28140	355.37						
80.....	3	28197	354.65	28	28198	354.64			
81.....	2	28267	353.77						
82.....	2	28336	352.91						
				29	28348	352.76	13	28350	352.7
83.....	2	28373	352.45						
84.....	2	28485	351.06						
85.....	2	28538	350.41	30	28540	350.38	14	28540	350.4
86.....	1	28590	349.77						
87.....	1	28658	348.94						
				31	28710	348.31			
							15	28990	344.9
							16	29210	342.4
							17	29750	336.1

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

PART OF THE REGION OF 6 μ

Hilger brass grating

Band No.	Wave-Length A	Wave No. per mm	Distance from Center, 159.6	Mean Distance of One Pair
1	50220	199.1	39.5	
2	50850	196.7	37.1	
3	51110	195.7	36.1	
4	51490	194.2	34.6	
5	51720	193.4	33.8	
6	52040	192.2	32.6	
7	52420	190.8	31.2	
7'	52600	190.1	30.5	
8	52800	189.4	29.8	
9	52960	188.8	29.2	
9'	53090	188.4	28.8	
10	53520	186.8	27.2	
11	54240	184.4	24.8	
11'	54470	183.6	24.0	
12	54660	182.9	23.3	
13	55250	181.0	21.4	
13'	55570	179.9	20.3	
14	55810	179.2	19.6	
15	56170	178.0	18.4	
16	56430	177.2	17.6	
17	56800	176.1	16.5	
17'	56920	175.7	16.1	
18	57170	174.9	15.3	
18'	57470	174.0	14.4	
19	57680	173.4	13.8	
20	58240	171.7	12.1	12.2
20'	58640	170.5	10.9	10.8
21	58830	170.0	10.4	
21'	58950	169.6	10.0	9.9
22	59380	168.4	8.8	8.8
22'	59720	167.5	7.9	7.9
23	59880	167.0	7.4	7.4
24	60160	166.2	6.6	6.4
25	60480	165.3	5.7	5.6
26	60720	164.7	5.1	
27	61140	163.6	4.0	3.9
27'	61440	162.8	3.2	3.2
28	61620	162.3	2.7	2.7
29	61880	161.6	2.0	2.0
30	62160	160.9		
30'	62420	160.2		
31	62720	159.4		
31'	62890	159.0		
32	63440	157.6	2.0	
33	63740	156.9	2.7	
33'	63930	156.4	3.2	
34	64180	155.8	3.8	
35	64530	155.0	4.6	
36	64950	154.0	5.6	
37	65200	153.4	6.2	
38	65510	152.6	7.0	

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

PART OF THE REGION OF 6 μ—*Concluded*

Hilger brass grating

Band No.	Wave-Length A	Wave No. per mm	Distance from Center, 159.6	Mean Distance of One Pair
39.........	65730	152.1	7.4	
40.........	65940	151.7	7.9	
41.........	66340	150.7	8.9	
42.........	66810	149.7	9.9	
43.........	67160	148.9	10.7	
44.........	67520	148.1	11.5	
45.........	67920	147.2	12.4	
46.........	68280	146.5	13.1	

V. QUESTIONS OF INTERPRETATION

In addition to her work with water-vapor mentioned above, Eva von Bahr[1] has studied the absorption of HCl in the neighborhood of 3.5 μ. The combination principle, with the quantum distribution of rotation-frequencies, has been applied by Bjerrum[2] to set forth reasons for the presence of these absorption bands of HCl and to calculate directly the size of energy quanta. However, the chaotic arrangement of water bands in the region of 2.6 μ, as they appear in Fig. 5, does not appear to satisfy the demands of these hypotheses, though the region has in a general way a center and a rough appearance of symmetry. Since this work was completed Mr. E. H. Imes has mapped the absorption of HCl at 3.5 μ with the apparatus here described, using an interval of about 30 A in the slit of the thermopile. The wave-lengths are precisely determined by the grating, and a parabolic shift of the vibration-frequency with increasing rotation-frequency is very definitely displayed. Work on HCl showing this effect has been published by Kemble and Brinsmade,[3] though that done here was completed before their work appeared. The curve of Fig. 5 does show a crowding together of bands in the nearer end, which our experience with HCl would lead us at least to consider possible. But the uncertainty, due to

[1] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **15**, 1150, 1913.

[2] *Ibid.*, **16**, 614, 1914.

[3] *Proceedings of the National Academy of Sciences*, **3**, 420, 1917.

the general complexity of the curve, as to which band should be associated with any other to make a pair, and about the location of the vibration center for no rotation, has so far deferred our explanation. The third rotation series mentioned in connection with the work of Rubens and Hettner may perhaps be appealed to as a probable cause of the complexity both at 2.6 and 6 μ.

The appearance of the curves, particularly of Fig. 5, suggests that useful knowledge may be still further advanced by an increase in dispersion. In the case of the 2.6 μ region, however, neither a finer nor a larger grating would serve this purpose; but a source of greater energy, or a more sensitive detector, would enable us to use narrower slits. We hope to profit by the use of a tungsten ribbon lamp in place of the glower. We may be able to build another more sensitive galvanometer.

It should be pointed out that the selective atmospheric absorption near 2.6 and 6 μ may account for the unexpected weakness or even the non-appearance of bright lines predicted by the laws of series in the spectra of elements.

It is hoped to employ the apparatus in the further study of absorption, and it may be that the behavior of carbon dioxide will help to explain that of water-vapor. The work here described was undertaken at the suggestion of Professor H. M. Randall, to whom the author makes grateful acknowledgment of his indebtedness.

University of Michigan, Physics Laboratory
September 25, 1917

Minor Contributions and Notes

A HELIUM STAR WITH LARGE PARALLAX, RADIAL VELOCITY (AND PROPER MOTION?)

Among the stars included in my program for parallax with the meridian circle and afterward selected for photographic determination of parallax was the star Boss *P.G.C.* 1517 = *A.G.C.* 7234 = 72 G Columbae, $\alpha = 6^h 0^m 37^s$, $\delta = -32° 10' 12''$ (1900), Harvard Mag. 5.6. This star, of type B, was observed at Professor Kapteyn's suggestion, who suspected a parallax of about $+0\overset{''}{.}1$, unusually large for this type.

The results obtained from the photographic determination are:

$$\pi = +0\overset{''}{.}069 \pm 0.006$$
$$\mu_\alpha = +0\overset{''}{.}235 = 0\overset{s}{.}0185$$

The parallax agrees very well with Professor Kapteyn's supposition. The proper motion in α, however, has come out much larger than that given in the Boss *P.G.C.*, which was $-0\overset{s}{.}0001$.

Mr. R. E. Wilson, of the D. O. Mills Observatory, was good enough to determine the star's radial velocity, which he found from three plates to be +102 km (that is, +83 km corrected for solar motion with $\alpha = 270°$, $\delta = +32°$, $V_0 = +19$ km). He remarked that the 6 lines measured were fairly broad, but that the radial velocity will not be more than 5 km in error. The radial velocity is a large one for a B-type star, so that the star must have a considerable space motion, though the transverse proper motion is still very uncertain.

The broad lines may indicate that it is a double star showing the lines of both components.

J. Voûte

Franschhack, South Africa
May 20, 1918